Mathematical Social Sciences 88 (2017) 55–65

Contents lists available at ScienceDirect

Mathematical Social Sciences

journal homepage: www.elsevier.com/locate/mss

A new index for measuring seasonality: A transportation cost approach

Giovanni L. Lo Magno [a], Mauro Ferrante [b,*], Stefano De Cantis [a]

[a] Department of Economics, Business and Statistics, University of Palermo, Viale delle Scienze — building 13, 90128 Palermo, Italy
[b] Department of Culture and Society, University of Palermo, Viale delle Scienze — building 15, 90128 Palermo, Italy

HIGHLIGHTS

- A novel approach for measuring seasonal amplitude is proposed.
- The approach is based on a transportation problem.
- The cyclic structure of time periods is taken into account.
- The proposed indices can be of interest in a wide range of fields.

ARTICLE INFO

Article history:
Received 4 May 2016
Received in revised form 6 May 2017
Accepted 7 May 2017
Available online 17 May 2017

Keywords:
Seasonal measurement
Seasonal index
Transportation problem

ABSTRACT

Seasonal fluctuations characterize many natural and social phenomena. Although the causes and impacts of seasonality are generally well documented in different study contexts, and many methods for isolating the seasonal component have been developed, considerably less attention has been paid to the measurement of the degree of seasonality. After reviewing the main indices used for measuring seasonality in different study contexts, we will propose a new approach in which seasonality is evaluated on the basis of the solution of a transportation problem. By considering the interdisciplinary nature of seasonal phenomena, the topic of measuring seasonality merits attention from a wide variety of perspectives.

1. Introduction

Seasonal variations characterize many natural and human phenomena. The word *seasonality* itself highlights the strong relationship between systematic intra-year variations and astronomical seasons. The different intensity of solar rays determines numerous effects on the environment, such as different levels of humidity and temperature, but also on animal behaviour (e.g. animals hibernating or migrating), and human habits (Ulijaszek and Strickland, 2009). The biological and environmental sciences (Wirth et al., 2001; Claereboudt et al., 1995), medicine (Rau, 2005), agricultural sciences (Gill, 1991), economics (Hylleberg, 1992b), social sciences (Hotelling and Hotelling, 1931; Ridderstaat and Nijkamp, 2015), engineering (Shi et al., 2015; Monson and Brezonik, 1998), and earth and planetary sciences (Liebmann et al., 2012) are all subject areas which, in some way, are concerned with the examination of seasonal phenomena.

Although the causes and impacts of seasonality are generally well documented in different study contexts and many methods for isolating the seasonal component have been developed, considerably less attention has been paid to the measurement of seasonality. Nevertheless, quantitative measurements are essential for comparing the degree of seasonality of different phenomena, or of the same phenomenon over different time periods or places. Thus, an appropriate measure of seasonality is required to improve our knowledge relating to seasonality; it should also take into account the timing which characterizes seasonal variations, as well as the ordinal and cyclical structure of the time periods during which the phenomenon is under observation.

Given these premises and having reviewed the main indices used in different study contexts, we will propose a new measurement approach in which seasonality is evaluated on the basis of the solution of a transportation problem (Hillier and Lieberman, 2014; Winston and Goldberg, 2004). This approach hinges on the problem of minimizing the cost of transferring units from high season time periods to low season periods in order to eliminate seasonality. Furthermore, it adopts a particular structure of transportation costs which takes into account the cyclical distance between time periods. Thus, two seasonality indices following this approach will be defined in this paper. A real data example will also be presented in order to show how the proposed approach captures various

* Corresponding author.
E-mail addresses: lomagno.gl@virgilio.it (G.L. Lo Magno), mauro.ferrante@unipa.it (M. Ferrante), stefano.decantis@unipa.it (S. De Cantis).

http://dx.doi.org/10.1016/j.mathsocsci.2017.05.002

compelling aspects of seasonality, which have been omitted by the majority of indices currently in use.

Nonetheless, we recognize that additional research is required in order to construct an axiomatic approach to the measurement of seasonality. Given the wide interest in the analysis of seasonal phenomena, the proposed measurement approach may have an inter-disciplinary application.

2. Measuring seasonality: main approaches and current limits

In general terms, seasonal variations of a time series are constituted by systematic though not necessarily regular intra-year movements in observed values, which appear every year with approximately the same periodicity (Thomas and Wallis, 1971). This periodicity can be attributed mainly to weather effects, connected to the seasons, and (in social and economic phenomena) to social uses which determine periodic behaviour (Vianelli, 1948) (or see the similar definition provided by Hylleberg (1992a, p. 4)). Seasonality is generally characterized by a well structured pattern rather than random irregularities, and seasonal fluctuations are determined by well-defined causes of different origin and intensity, depending on the study context (Granger, 1979).

When the period being analysed is relatively long, the trend-cycle component (i.e. medium–long term tendency of the series), which generally affects time series behaviour, makes unreliable a direct comparison of seasonal fluctuations (i.e. infra-annual variations). Therefore, it is common practice – when analysing and measuring seasonal fluctuations – to decompose the series into a trend-cycle, a seasonal and an irregular/random component (Dagum, 2004). The notion that a time series is composed of several components with different periodicity has a long tradition. It can be traced back to the studies of astronomers in the seventeenth-century and the studies of the mathematician and physicist Joseph Fourier in the early 1800s.

Since then, much of the attention paid to time series analysis has focused on methods of seasonal adjustment, aimed at decomposing a time series into several elements in order to isolate the seasonal component. Several methods for seasonal adjustment have been proposed and these can be classified into three main groups: regression methods, empirical filtering and signal extraction (Planas, 1998). Referring to more specialized texts regarding the analytical properties of univariate time series processes (Chatfield, 2013; Hamilton, 1994), these methods generally allow for the determination of seasonal factors (Dagum, 1975), which should represent only the seasonal component of the series. Seasonal factors are usually centred on the value 1 (or 100), which would indicate a multiplicative seasonal component in line with the trend-cycle component. Values greater (or smaller) than 1 would indicate a seasonal component above (or below) the trend-cycle component.

Let $\{z_{t,i}\}$ represent the seasonal factors series, which can be derived according to one of the aforementioned approaches, where $i = 1, 2, \ldots, I$ represents the year and t represents the period within the year, which is $t = 1, 2, \ldots, 12$ (for example) in the case of a monthly series. Table 1 presents and classifies the most frequent seasonality measures, according to the aspect of seasonality these measures aim to evaluate. Each of the proposed measures can synthesize different aspects of seasonality. According to Kuznets (1932), it is possible to focus on several aspects of seasonality, such as: (a) the pattern of seasonal swing, that is, the distribution of seasonal factors within the months of a given year; (b) the amplitude of seasonal swing through a measure of synthesis of seasonal factors in a given year; (c) the persistency or the variations in seasonal patterns through various measures of the variability of the seasonal pattern over several years; and (d) the persistency or the variations in seasonal amplitude, through some measures of variability of the seasonal amplitude covering several years.

Pattern measures can be derived from seasonal indices, given by the mean of seasonal factors in each month over all the years considered (Falkner, 1924). They allow for the analysis of the series pattern (on average, during all the years under consideration) and they can be used to define high and low seasons (months for which seasonal indices are above or below the trend-cycle component). However, the reliability of seasonal indices is related to the values of the coefficient of variability associated with each seasonal index, which is in turn a measure of the variability in seasonal factors for a given month, for all the years under consideration. Low values of the coefficients of variability would indicate a persistency or stability in the seasonality pattern, and this would be converse for high values (Falkner, 1924). These coefficients permit us to obtain information regarding the relative pattern stability over the different years under consideration. Seasonal peak, seasonal ratio, and seasonal range provide information relating to the general degree of imbalance among seasonal factors (Bell and Goring, 1998; Lim and McAleer, 2001; Pardo et al., 2002), in absolute (seasonal peak, seasonal range) or relative (seasonal ratio) terms. However and as pointed out by Wanhill (1980) in the field of tourism, these measures only take into account the extreme values of seasonal factors (of each year). This means that all three measures do not include any information regarding the other values in the annual series.

Another simple index for seasonality, used in Demography, is the winter/summer ratio (Rau, 2005), in which the mean of seasonal factors in winter months is divided by the mean of seasonal factors occurring in summer months. Such an index can provide information relating to pattern and amplitude of seasonality by comparing the summer and winter seasons. Thus, it provides us with a measurement of the differential between the two seasons, but it does not offer information as to what happens in other months (Rau, 2005). Moreover, when considering the mean of seasonal factors in certain periods (e.g. winter and summer months), the winter/summer ratio does not take into account the entire distribution of seasonal factors. Values lower than 1 would indicate a prevalence of the phenomenon of interest in summer months compared to winter months, and conversely for values greater than 1. Moreover, this index could be modified in order to compare different seasons of the year (e.g. spring/autumn, summer/spring, etc.).

The coefficient of seasonal variation (Koenig-Lewis and Bischoff, 2005) provides information regarding the variability of seasonal factors in a given year. Since it does not take into account the cyclical order of months, it is unable to provide any information on the seasonality pattern. Consequently, very different seasonal patterns could produce very similar values of seasonal amplitude (De Cantis et al., 2011). The same problem occurs with one of the most commonly used seasonality indices: the Gini concentration index (Gini, 1912). This was originally proposed as a measure of economic inequality and it is employed here as a measure of inequality between seasonal factors in any given year (Cisneros-Martínez and Fernández-Morales, 2015; Fernández-Morales et al., 2016; Fernández-Morales, 2003; Fernández-Morales and Mayorga-Toledano, 2008; Groves-Kirkby et al., 2009; Lee, 1996; Wanhill, 1980). The same considerations could be made with reference to other measures used for seasonality, such as the *Theil index* (Fernández-Morales, 2003; Duro, 2016), the *Precipitation Concentration Index* (Oliver, 1980; Apaydin et al., 2006), and the *Modified Fournier Index* (Arnoldus, 1980; Apaydin et al., 2006), all of which do not take into account the natural ordering of months. For example, two situations, one in which the total amount of the phenomenon of interest is equally concentrated into two contiguous months (e.g. July and August), and the other in