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The Pareto principle and resource egalitarianism

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HIGHLIGHTS

- I partially characterize the consensual Rawlsian social ordering (Sprumont 2012).
- I introduce and characterize the leximin Paretian ordering.
- I propose a strong bundle-reducing principle and a strong Permutation Pareto axiom.
- I prove that these two axioms are mutually compatible.
- I show that the leximin Paretian ordering satisfies these axioms.

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ABSTRACT

This paper introduces and studies the *leximin Paretian ordering*, which refines the consensual leximin ordering by adding the Pareto principle to the concept of lexicographic egalitarianism. We also provide an alternative characterization of the consensual Rawlsian ordering. We introduce several new axioms, including the Permutation Pareto Principle and Internal Dominance, and study their logical relationships.

1. Introduction

In this paper, we define and study egalitarianism in the context of social ordering and in a multi-commodity model with a fixed number of agents. Social ordering addresses the question of how we specify the aim of the society which aggregates individual preferences to construct an ordering over all conceivable allocations. Egalitarianism in this paper does not necessarily mean distributing all the commodities equally to all the agents. It does mean that unequal allocations should be based on preferences, not on endowments or political advantage.

The study of egalitarianism started in the context of a single commodity. Even though equal division is desirable under unidimensional egalitarianism, it is useful to establish notions to rank the extent of inequality. Sen (1973) establishes the weak equity axiom as an egalitarian notion in the welfare economics context, which requires that an individual who is less able to transfer income relative to another should receive more income. Hammond (1976) captures this idea by introducing the equity axiom in the social choice context,[1] and shows that the only class of social orderings that satisfies the equity axiom, Pareto principle, and the symmetry principle (if permutation of incomes makes everyone indifferent, then the two allocations are socially indifferent) is the leximin ordering.[2] Hardy et al. (1934) show the equivalence between Lorenz dominance and being dominant by a finite sequence of Pigou–Dalton transfers (Pigou, 1912; Dalton, 1920).

A number of current studies, however, have examined a multidimensional context, and researchers generally agree that the multicommodity model cannot be represented by a single dimension. Kolm (1977) first studies the multidimensional dominance issue in the welfare economics context. Marshall and Olkin (1979), however, stress that it is fairly difficult to extend the results of unidimensional models to more dimensions. Fleurbaey and Trannoy (2003) also point out that the standard weak Pareto principle is incompatible with the bundle-reducing transfer principle[3] when agents possibly have different preferences over bundles.[4] In a multicommodity model, Fleurbaey (2005, 2007) and Fleurbaey and

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[1] The equity axiom (Hammond, 1976) argues that for any allocations x, y and any agents i, j, if j benefits more than i in both x and y, and if j prefers y to x while i prefers x to y and all the other agents are indifferent between x and y, then y should be weakly preferred to x by society.

[2] Hammond (1976) refers to this family of social orderings as the lexical difference principle.

[3] The bundle-reducing transfer principle says that if $(x_1, \ldots, x_n)$ is a multidimensional allocation, $(x_1, \ldots, x_i - t, \ldots, x_j + t, \ldots, x_n)$ is better than $(x_1, \ldots, x_n)$ whenever $x_j < x_j + t \leq x_i - t < x_i$.

[4] This analysis is in line with Sen (1970), who shows in the unidimensional social choice context that Liberalism (for each agent i, there is at least one pair of allocations x, y such that $y \succeq_i x$ if and only if y is weakly preferred to x by the society) is incompatible with the Pareto principle.

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Maniquet (2008) study weaker notions of the bundle-reducing transfer principle compatible with the Pareto principle. Sprumont (2012) proposes Consensus, a weaker axiom than the Pareto principle which says that if for any allocations x and y everyone agrees that everyone's bundle at y is strictly better than that at x then y is strictly preferred to x, which is compatible with Dominance Aversion, even a stronger axiom than bundle-reducing transfer principle which states that if $(x_1, \ldots, x_n)$ is a multidimensional allocation, $(y_1, y_2, x_3, \ldots, x_n)$ is socially preferred to $(x_1, \ldots, x_n)$ if $x_1 < y_1 \leq y_2 < x_2$.

Sprumont (2012) proposes the leximin social ordering as a foundation of consensual egalitarianism. It satisfies both Dominance Aversion and Consensus. However, as Sprumont (2012) points out, this social ordering fails to capture Pareto dominance between certain pairs of allocations. To be specific, it is reasonable to argue that even a social planner who considers the leximin order as a decisive standard may try to Pareto dominate the allocation by permuting bundles,[5] which does not change the leximin ranking of the allocation. The main goal of this paper is to improve on Sprumont (2012) by applying Permutation Pareto principle[6] as an alternative weakened version of the Pareto principle to Consensus, and applying Strong Dominance Aversion which states that if $(x_1, \ldots, x_n)$ is a multidimensional allocation, $(y_1, y_2, x_3, \ldots, x_n)$ is socially weakly preferred to $(x_1, \ldots, x_n)$ if $x_1 < y_1, y_2 < x_2$, and y_2 is socially weakly preferred to y_1.[7] That is, we show that stronger notions of Paretian axioms and the bundle-reducing transfer principle than those used in Sprumont (2012) are applicable.

We propose the leximin Paretian ordering, a social ordering using the leximin ordering as the most important criterion, and considering Permutation Pareto dominance as a secondary standard. In other words, this social ordering endeavors to achieve utilitarianism while obeying the egalitarianism that other social orderings pursue, in the framework of this paper. We justify the leximin Paretian ordering with several axioms.

This paper is organized as follows. Section 2 introduces the model and conditions. Section 3 lays out the axioms and the main results. We provide proofs of propositions and a discussion of the independence of the axioms in the main proposition in an Appendix.

2. Preliminaries

Let there be a fixed number of commodities $m \geq 2$ in the economy. Let $X = \mathbb{R}^m_+$ be the commodity space. Let $N = \{1, \ldots, n\}$ represent a fixed, finite set of individuals such that $n \geq 2$, and let X^N be the set of conceivable allocations. For any bundle $a \in X$, allocation $x \in X^N$, $1 \leq j \leq m$, and $i \in N$, a_j represents the quantity of the jth commodity, x_i represents the ith bundle in x, and x_{ij} represents the quantity of the ith bundle's jth commodity in x. For any $a, b \in X$, $a \geq b$ if and only if $a_i \geq b_i$ for all i, $a > b$ if and only if $a_i \geq b_i$ for all i and the inequality is strict for at least one person. Every agent $i \in N$ has a strictly monotonic,[8] continuous,[9] and rational[10] preference ordering R_i over X. This paper aims to set a social ordering $\mathbf{R}$ over X^N. Let P_i denote the strict preference relation associated with R_i, and $\mathbf{P}$ denote the strict preference relation associated with $\mathbf{R}$.

We say R is an ordering over X which agrees with $\cap_{i\in N} R_i$ when for any two bundles $a, b \in X$, bP_ia for all $i \in N$ implies bPa. Note that bR_ia for all $i \in N$ also implies bRa for any $a, b \in X$ if R is continuous since each R_i for any $i \in N$ is strictly monotonic. The term 'agree with $\cap_{i\in N} R_i$' was used in Sprumont (2012), which expresses 'society's evaluation' of the relative value of commodity bundles.

For any allocation $x = (x_1, \ldots, x_n) \in X^N$, denote by $(x^R_1, \ldots, x^R_n)$ the allocation obtained by rearranging the bundles $x_1, \ldots, x_n$ from the worst to the best so that agent 1 now has the worst and agent n has the best according to R, that is, $x^R_n R x^R_{n-1} R \cdots R x^R_1$ with a tie-breaking rule as follows: for any x_i and x_j such that $i < j$ and x_iIx_j, x_i is arranged before x_j in $(x^R_1, \ldots, x^R_n)$. We denote $x^R = (x^R_1, \ldots, x^R_n)$.

For any $x \in X^N$ and any permutation π on N, we denote $\pi(x) = (x_{\pi(1)}, \ldots, x_{\pi(n)}) \in X^N$. For any $x, y \in X^N$, we say y is *permuted* from x with a permutation π on N if $y = \pi(x)$. We also define *Pareto dominance* in a formal way: For any $x, y \in X^N$, we say that y *Pareto dominates* x, denoted $y \geq_{par} x$, when $y_iR_ix_i$ for all $i \in N$, and say that y *strictly Pareto dominates* x, denoted $y >_{par} x$, when $y_iR_ix_i$ for all $i \in N$ and $y_jP_jx_j$ for at least one $j \in N$. We also say that there is a Pareto dominance between x and y if $x \geq_{par} y$ or $y \geq_{par} x$.

3. Lexicographic egalitarianism and the Pareto principle

This section studies classes of social orderings and introduces axioms to provide characterization results.

3.1. Consensual Rawlsian ordering

Sprumont (2012) studies a notion of egalitarianism that comes from Rawls (1971) by introducing a social ordering.

Definition 1. A social ordering $\mathbf{R}$ is a *consensual Rawlsian ordering* if and only if there is a continuous ordering R on X agreeing with $\cap_{i\in N} R_i$ such that, for all allocations $x, y \in X^N$, $y\mathbf{P}x$ if $y^R_1 P x^R_1$.

We adopt a weak notion of continuity and the Paretian axiom, namely Weak Continuity and Consensus from Sprumont (2012), as the two basic, plausible concepts for society to pursue.

Even though lexicographic social orderings are not continuous (see, for example, Mas-Colell et al., 1995, p. 47 Example 3.C.1.), they do satisfy a weak form of continuity. *Weak Continuity* requires the social ordering be continuous only for 'fully egalitarian' allocations.

Weak continuity. For any $a, b \in X$ and any sequence $\{b^k\}$ in X converging to b, $(b^k, \ldots, b^k)\mathbf{R}(a, \ldots, a)$ for all k implies $(b, \ldots, b)\mathbf{R}(a, \ldots, a)$.

Weak Continuity is desirable for social orderings given that all the individual preferences are continuous.

Consensus is a Paretian axiom weaker than the standard Pareto principle. Consensus says that an allocation is preferred to another allocation if all the individuals prefer every bundle in the former allocation to that in the latter.

Consensus. For any $x, y \in X^N$, if $y_iP_jx_i$ for all $i, j \in N$, then $y\mathbf{P}x$.

Dominance Aversion (Sprumont, 2012) is an egalitarian notion which says that reducing bundle dominance is always desirable.

5 Suppes (1966) first applies permutation criterion in a unidimensional context to introduce the Suppes grading principle, and Saposnik (1981) applies the Suppes principle to the multicommodity context.

6 The Permutation Pareto Principle states that if permuting bundles can result in every agent preferring her new bundle to the old, then this new allocation should be considered as better.

7 Strong Dominance Aversion is stronger than Dominance Aversion given that Consensus is satisfied. See Section 3 for details.

8 The preference R_i is *monotonic* if for any $a, b \in X$, $b \geq a$ and $b_j > a_j$ for some j implies bR_ia.

9 The preference R_i is *continuous* if for any sequence of pairs (a^k, b^k) where $a^k, b^k \in X$ and $b^kR_ia^k$ for all k, and $a^k \to a$ and $b^k \to b$, bR_ia.

10 The preference R_i is *complete* if for all $a, b \in X$ either aR_ib, bR_ia, or both are true, R_i is *transitive* if for all $a, b, c \in X$ aR_ib and bR_ic implies aR_ic, and R_i is *rational* if it is complete and transitive.