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Moments expansion densities for quantifying financial risk

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ABSTRACT

We propose a novel semi-nonparametric distribution that is feasibly parameterized to represent the non-Gaussianities of the asset return distributions. Our Moments Expansion (ME) density presents gains in simplicity attributable to its innovative polynomials, which are defined by the difference between the n*th* power of the random variable and the n*th* moment of the density used as the basis. We show that the Gram–Charlier distribution is a particular case of the ME-type of densities. The latter being more tractable and easier to implement when quadratic transformations are used to ensure positiveness. In an empirical application to asset returns, the ME model outperforms both standard and non-Gaussian GARCH models along several risk forecasting dimensions.

1. Introduction

The low performance of many traditional methods for financial risk modeling and forecasting during the recent credit crunch has highlighted the deficiencies of standard models for capturing the high-order moments and salient stylized regularities of the asset return distributions (Cont, 2001). Generalized autoregressive conditional heteroscedasticity (GARCH) models (Bollerslev, 1986; Engle, 1982), comprehensively reviewed in Terasvirta (2009), have been extended to account for non-Gaussianities. The alternative densities that have been proposed in previous studies for that purpose include: (i) parametric probability density functions (pdfs henceforth), for instance, the standardized Student's t (Bollerslev, 1987), the GED (Nelson, 1991), the skewed t (Hansen, 1994), the normal inverse Gaussian (NIG) (Jensen & Lunde, 2001), mixtures of normals (Alexander & Lazar, 2006), BEGE combination of gammas distribution (Bekaert, Engstrom, & Ermolov, 2014), Variance-Gamma (Göncü & Yang, 2016); (ii) non-parametric pdfs (Engle & Gonzalez-Rivera, 1991); and (iii) semi-nonparametric (SNP) densities based on Gram–Charlier (GC) series expansions (Charlier, 1905), introduced in econometrics by Sargan (1976) and developed by authors such as, Gallant and Nychka (1987), Mauleón and Perote (2000), León, Mencía, and Sentana (2009), among others.

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The current global financial market scenario requires that any assumed density for the shocks in GARCH-type models is flexible enough to account for the leptokurtosis and multimodality of the empirical asset return distributions. In this respect, it is well known that heavy-tailed parametric pdfs overestimate the frequencies for mid-to-lower quantiles, as they try to capture the accumulation of observations in the distribution tail through a monotonic decay. On the other hand, non-parametric densities are more flexible to fit jumps in the distribution tail but they require very large data sets in order to achieve a reasonable degree of precision. Alternatively, SNP pdfs, characterized by its flexibility to represent any frequency function at any degree of accuracy (Cramér, 1925), are capable of fitting the wavy shape of the return distributions tail, exacerbated by periods of high financial instability. This fact has awakened a renewed interest for SNP methods and their applications for measuring financial risk– see, e.g., Huang, Lin, Wang, and Chiu (2014), Lin, Huang, and Li (2015), Ñíguez and Perote (2016) or León and Moreno (2017). SNP methods, however, also have well-known drawbacks:

(i) Truncated SNP functions (i.e. finite series expansions) are not really pdfs since they may yield negative values (Barton & Dennis, 1952). This issue has been addressed through either parametric constraints à la Jondeau and Rockinger (2001), or density reformulations based on the methodology of Gallant and Nychka (1987) and Gallant and Tauchen (1989) (GNT hereafter).
(ii) Complexity: (a) The direct interpretation of moments in terms of the density parameters is lost when GNT transformations are applied; (b) the characterizations of the density in terms of either the cumulative distribution function (cdf) or the moments generating function (mgf) for GNT-GC pdfs are difficult to obtain; and (c) maximum likelihood (ML) suboptimization is likely to occur.
(iii) SNP pdfs are sensitive to choices in the number of expansion terms.

In this study we present a SNP pdf that, preserving the flexibility typical of GC pdfs, allows to addressing the aforementioned complexities. To do so, we introduce an original series expansion, whose terms are defined as the difference between the n*th* power of the variable and the n*th* moment of the parametric density used as the basis of the expansion. Our Moments Expansion density (ME henceforth) presents gains in simplicity that ease both its theoretically analysis, and practical implementation to model high-order moments and risk measures. We show that the ME is a general family of distributions that nests the GC when the Gaussian density is taken as the basis and GNT transformations are not implemented.

In an empirical application to asset returns, we test the applicability of our model in terms of its relative performance for multiperiod density forecasting as well as for predicting overall measures of market risk, such as, volatility and Value-at-Risk (VaR). The alternative distributions we consider are: Gaussian (used as framework); standardized Student's t; symmetric and skewed GC; and NIG.[1] The models ability for forecasting the conditional variance is measured through ranking-robust loss functions for imperfect volatility proxies (Patton, 2010). VaR forecasting accuracy is assessed through a battery of tests, namely, the magnitude of exceptions statistic (López, 1999), likelihood ratio (LR) tests (Christoffersen, 1998), and the HIT test of Engle and Manganelli (2004). For multiperiod density forecasting we follow the methodology proposed by Maheu and McCurdy (2011). Our results show that Gaussian-ME models are capable of outperforming both standard and non-Gaussian GARCH models for conditional density forecasting, as well as, along several dimensions of market risk forecasting.

The remainder of the paper is organized as follows. In Section 2, we define the ME pdf and analyze its statistical properties. Section 3 provides a comparative analysis of the ME model for forecasting density and financial risk. In Section 4, we summarize our conclusions. All of the proofs are provided in the Appendix A.

2. The ME density

In this section, we define the ME pdf and study its characterizations and statistical properties in relation to the GC, which we use as framework.

Let a random variable x be GC (Type A) distributed with pdf given by,

$$\pi(x, \mathbf{d}_n) = \left(1 + \sum_{s=1}^{n} d_s H_s(x)\right)\phi(x), \tag{1}$$

where $\phi(\cdot)$ stands for the standard Normal pdf, $H_s(\cdot)$ denotes the Hermite polynomial (HP) of order s, and $d_n = (d_1, d_2, \ldots, d_n)' \in \mathbb{R}^n$ with n being the truncation order of the expansion. The HPs, which can be defined as in Eq. (2) form an orthogonal basis with respect to $\phi(x)$, which is the grounds for $\pi(x, d_n)$ to integrate up to one.

$$H_s(x) = \frac{(-1)^s}{\phi(x)} \frac{d^s \phi(x)}{dx^s}. \tag{2}$$

[1] Other interesting parametric densities to consider include the skewed Student's t; see Ergen (2014) for a recent study of this density for VaR forecasting.