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Numerical analysis of steady thermo-elastic wear regimes induced by translating and rotating punches

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ABSTRACT

The present work provides the analysis of coupled thermo-elastic steady wear regimes: wear analysis of a punch translating on an elastic strip and wear induced by a rotating punch on a toroidal surface. The contact pressure and temperature are specified from the stationary conditions of the wear dissipation power or from the contact-conformity-condition. The wear and friction parameters are assumed as fixed or temperature dependent. Three transverse friction models are discussed for wear debris motion. The analysis and results presented can be used in design of optimal contact shapes assuring the steady wear regimes throughout the whole contact operation period.

1. Introduction

The present paper provides the numerical analysis of steady wear regimes induced by relative sliding of two contacting bodies B_1 and B_2 and follows the previous works by Páczelt and Mróz [1–4]. It was shown that the stationary conditions of the global wear dissipation power on the contact surface S_c provide the pressure distribution. The shape of contacting bodies on S_c associated with this distribution is next numerically specified. This approach provides the possibility to predict the wear rate and contact shape evolution in the steady state. A related optimal shape design problem of contacting bodies can then be considered by postulating the steady state regime throughout the whole contact operation period, thus avoiding transient run-in periods of larger wear rates.

The coupled temperature field due to external boundary conditions or heat generation due to friction and wear induces thermal distortion and essentially affects the steady state shape of contact surface, however, the pressure distribution on S_c remains unchanged. This class of problems was analyzed in Páczelt and Mróz [4] by applying the coupled thermo-mechanical approach and the minimum principle of the global wear dissipation power. The cases of drum and disk brakes were analyzed in detail.

In the present paper the analysis of thermoelastic wear regimes in relative contact translation and rotation of two bodies is presented.

Body B_1 is treated as a punch executing the relative sliding motion on the substrate B_2. The stress and temperature fields in the steady state are specified together with the shape of contact surface reached due to wear process. These two relative sliding modes are most important for engineering applications.

The paper is organized as follows. In Section 2 the wear rule and the steady state conditions are presented. Three cases are distinguished, namely the wear of punch, the wear of two bodies and the wear of substrate. The steady state conditions are formulated for each case. In Section 3 the thermo-mechanical analysis problem is formulated and the numerical procedure is described for coupled thermal and mechanical fields following the previous work [4]. The contact pressure distribution was specified for cases when wear parameters and coefficient of friction are constant or temperature dependent. In the variational form of thermal equations the Petrov–Galerkin upwinding term [5,6] is introduced in order to avoid oscillatory solutions usually occurring for large sliding velocities and discontinuous boundary conditions. The numerical analysis of wear process induced by sliding punch on a flat substrate is presented in Section 4. The cases of wear of punch only and of combined wear of punch and substrate are treated numerically assuming the steady state condition requiring the uniform contact pressure distribution. The case of wear of substrate only is analyzed numerically by applying the contact conformity condition. In Section 5 the general analysis of a rotating punch of an arbitrary axisymmetric shape on a fixed substrate is analyzed. Three generalized transverse friction models are introduced with account for wear debris motion within the contact interface layer. The stationary conditions of minimization of the global wear dissipation

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power are used to derive contact pressure distribution with account for the equilibrium constraint and the respective friction rule. The general formulae are derived for contact pressure distribution which provides the input for iterative specification of optimal shape of contact surface. The optimal contact surface shape is specified and the sensitivity with respect to friction model, geometric and material parameters is analyzed. It is believed that the presented numerical solutions of specific cases illustrate the general methodology of analysis of steady wear states and specification of optimal contact shapes which can be applied in engineering design.

2. Wear rule and steady state conditions

2.1. Wear rule

Consider the contact problem of two elastic bodies B_α (α = 1, 2). The bodies B_1 and B_2 are assumed to undergo a relative sliding motion on the potential contact surfaces $S_c^{(1)}$ and $S_c^{(2)}$. The typical case occurs when body B_1 plays the role of punch executing the relative sliding motion on the substrate body B_2. When a punch of conforming contact zone S_c slides on a substrate and the zone is fixed with respect to material points of B_1, then only contact surface shape evolution occurs due to wear process. On the other hand, for a non-conforming contact surface of B_1, the wear process induces variation of both contact surface shape and contact zone size. The contact zone S_c then slides along the potential contact surface $S_c^{(2)}$ of body B_2 (e.g. plane punch, or ball sliding on a substrate). For some special cases the potential contact surfaces of two bodies are the same and coincide with the actual contact zone S_c. A most general case occurs when the contact zone S_c moves with respect to both bodies B_1 and B_2 and the wear process occurs due to periodic motion of S_c. (e.g. wear of gear teeth or roller bearings).

The contact stress components are denoted by p_n and $\boldsymbol{\tau}_n$ where p_n is the contact pressure and τ_n denotes the shear stress oriented along the relative sliding velocity vector $\boldsymbol{v}_r = \dot{\boldsymbol{u}}_\tau = \dot{\boldsymbol{u}}_\tau^{(2)} - \dot{\boldsymbol{u}}_\tau^{(1)}$, where $\dot{\boldsymbol{u}}_\tau^{(1)}, \dot{\boldsymbol{u}}_\tau^{(2)}$ are the tangential velocity vectors on S_c, see Fig. 1. The Signorini contact conditions are valid in the normal direction. In the tangential contact direction the Coulomb friction condition and the slip rules apply: $\tau_n = \mu p_n, \frac{\boldsymbol{\tau}_n}{\|\boldsymbol{\tau}_n\|} = \frac{\boldsymbol{v}_r}{\|\boldsymbol{v}_r\|}$. The relative sliding velocity is composed of elastic and rigid body terms

$$\begin{aligned} &\dot{\boldsymbol{u}}_\tau = \dot{\boldsymbol{u}}_{e,\tau}^{(2)} + \dot{\boldsymbol{u}}_{R,\tau}^{(2)} - (\dot{\boldsymbol{u}}_{e,\tau}^{(1)} + \dot{\boldsymbol{u}}_{R,\tau}^{(1)}) = \dot{\boldsymbol{u}}_{e,\tau} + \dot{\boldsymbol{u}}_{R,\tau}^{(s)}, \quad \|\dot{\boldsymbol{u}}_\tau\| = v_r, \\ &\dot{\boldsymbol{u}}_{R,\tau}^{(s)} = \dot{\boldsymbol{u}}_R^{(s)} + \boldsymbol{\Omega}_R^{(s)} \times \Delta\boldsymbol{r}, \end{aligned} \tag{1}$$

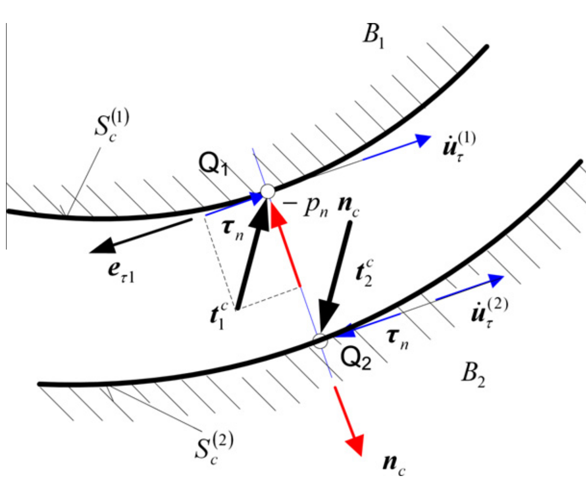

Fig. 1. Contact stress $\boldsymbol{t}_i^c,\ i = 1, 2$, tangential velocities $\dot{\boldsymbol{u}}_\tau^{(i)},\ i = 1, 2$, relative velocity $v_r = \left\|\dot{\boldsymbol{u}}_\tau^{(2)} - \dot{\boldsymbol{u}}_\tau^{(1)}\right\|, \dot{\boldsymbol{u}}_\tau = \dot{\boldsymbol{u}}_\tau^{(2)} - \dot{\boldsymbol{u}}_\tau^{(1)} = -v_r \boldsymbol{e}_{\tau 1}$.

where $\dot{\boldsymbol{u}}_{R,\tau}^{(s)}$ is the rigid body sliding velocity, $\dot{\boldsymbol{u}}_R^{(s)}$ and $\boldsymbol{\Omega}_R^{(s)}$ are the relative sliding and rotation velocities. The wear velocity is associated with the rigid body motion induced by wear

$$\dot{\boldsymbol{u}}_R^{(w)} = \dot{\boldsymbol{\lambda}}_F + \dot{\boldsymbol{\lambda}}_M \times \Delta\boldsymbol{r}, \tag{2}$$

where $\dot{\boldsymbol{\lambda}}_F$ and $\dot{\boldsymbol{\lambda}}_M$ are the relative translation and rotation velocities induced by wear and $\Delta\boldsymbol{r}$ is the position vector with respect to a reference point.

Let us note that the wear velocity, vectors $\dot{\boldsymbol{\lambda}}_F$ and $\dot{\boldsymbol{\lambda}}_M$ should be determined from the solution of a specific problem, but the sliding velocity components are specified from the boundary conditions. In the analysis of sliding wear problems the elastic term of relative sliding velocity is usually neglected.

Assume the isotropic wear rule in the form [1,2]

$$\dot{w}_{i,n} = \beta_i(\tau_n)^{b_i} \|\dot{\mathbf{u}}_\tau\|^{a_i} = \beta_i(\mu p_n)^{b_i} v_r^{a_i} = \tilde{\beta}_i p_n^{b_i} v_r^{a_i}, \quad i = 1, 2, \tag{3}$$

where the material parameters $\tilde{\beta}_i$, a_i, b_i specify the wear rates of two contacting bodies, $\tilde{\beta}_i = \beta_i \mu^{b_i}$, μ is the coefficient of friction in sliding direction, $v_r = \|\dot{\boldsymbol{u}}_\tau\|$ is the relative velocity between two bodies. For dimensional convenience the contact pressure and the sliding velocity are referred to the nominal values p_0 = 1 MPa, v_0 = 1 m/s. When $a_i = b_i = 1$, the familiar Archard wear rule is obtained. The wear rule (3) specifies the wear rate $\dot{w}_{i,n}$ of the ith body in the normal contact direction. However, in general contact conditions the wear rate vector $\dot{\boldsymbol{w}}_i$ is not normal to the contact surface and results from the constraints imposed on the rigid body motion of punch B_1. Introducing the local reference triad $\boldsymbol{e}_{\tau 1}$, $\boldsymbol{e}_{\tau 2}$, $\boldsymbol{n}_c$ on the contact surface S_c, where $\boldsymbol{n}_c$ is the unit normal vector, directed into body B_2, $\boldsymbol{e}_{\tau 1}$ is the unit tangent vector coaxial with the sliding velocity and $\boldsymbol{e}_{\tau 2}$ is the transverse unit vector, the wear rate vectors of bodies B_1 and B_2 are

$$\begin{aligned} \dot{\boldsymbol{w}}_1 &= -\dot{w}_{1,n}\boldsymbol{n}_c + \dot{w}_{1,\tau 1}\boldsymbol{e}_{\tau 1} + \dot{w}_{1,\tau 2}\boldsymbol{e}_{\tau 2}, \\ \dot{\boldsymbol{w}}_2 &= \dot{w}_{2,n}\boldsymbol{n}_c - \dot{w}_{2,\tau 1}\boldsymbol{e}_{\tau 1} - \dot{w}_{2,\tau 2}\boldsymbol{e}_{\tau 2}, \end{aligned} \tag{4}$$

and the contact traction on S_c can be expressed as follows

$$\boldsymbol{t}^c = \boldsymbol{t}_1^c = -\boldsymbol{t}_2^c = -p_n \boldsymbol{\rho}_c^{\pm}, \quad \boldsymbol{\rho}_c^{\pm} = \boldsymbol{n}_c \pm \mu \boldsymbol{e}_{\tau 1} - \mu_d \boldsymbol{e}_{\tau 2}, \tag{5}$$

where $\boldsymbol{\rho}_c^{\pm}$ specifies the orientation and magnitude of traction $\boldsymbol{t}^c$ with reference to the contact pressure p_n and μ_d is the transverse friction coefficient. The sign + in (5) corresponds to the case when the relative velocity is $\dot{\boldsymbol{u}}_\tau = \dot{\boldsymbol{u}}_\tau^{(2)} - \dot{\boldsymbol{u}}_\tau^{(1)} = -\|\dot{\boldsymbol{u}}_\tau\|\boldsymbol{e}_{\tau 1} = -v_r \boldsymbol{e}_{\tau 1}$ with the corresponding shear stress acting on the body B_1 along $-\boldsymbol{e}_{\tau 1}$.

2.2. Steady state conditions

The wear analysis presented in this paper will be referred to the conforming punch contact with the substrate body B_2. The contact zone S_c is then specified and the wear process induces contact surface shape evolution of both bodies along the sliding path. Three cases can be distinguished, namely: wear of punch only, wear of substrate only and combined wear of both bodies.

Consider first the case of punch wear in sliding motion along the surface $S_c^{(2)}$. The fundamental coaxiality rule was stated by Páczelt and Mróz [3,4], namely: in the steady state the wear rate vector $\dot{\boldsymbol{w}}_R$ is collinear with the rigid body wear velocity vector $\dot{\boldsymbol{\lambda}}_R$, thus

$$\dot{\boldsymbol{w}}_R = \dot{w}_R \boldsymbol{e}_R, \quad \boldsymbol{e}_R = \frac{\dot{\boldsymbol{\lambda}}_R}{\|\dot{\boldsymbol{\lambda}}_R\|} = \frac{\dot{\boldsymbol{\lambda}}_F + \dot{\boldsymbol{\lambda}}_M \times \Delta\boldsymbol{r}}{\|\dot{\boldsymbol{\lambda}}_F + \dot{\boldsymbol{\lambda}}_M \times \Delta\boldsymbol{r}\|} \tag{6}$$

The coaxiality rule is illustrated in Fig. 2, where the local reference frame $\boldsymbol{e}_{\tau 1}$, $\boldsymbol{e}_{\tau 2}$, $\boldsymbol{n}_c$ is shown. The normal and tangential wear rate components now are

$$\dot{w}_n = \dot{w}_R \cos\chi, \quad \dot{w}_\tau = \dot{w}_R \sin\chi = \dot{w}_n \tan\chi \tag{7}$$

where χ is the angle between $\boldsymbol{n}_c$ and $\boldsymbol{e}_R$. For $\dot{\boldsymbol{\lambda}}_M = 0$ only uniform punch translation velocity $\dot{\boldsymbol{\lambda}}_F$ is allowed due to wear and for plane