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Campbell and Cochrane meet Melino and Yang: Reverse engineering the surplus ratio in a Mehra–Prescott economy ☆

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ABSTRACT

The well-known habit model of Campbell and Cochrane (1999) specifies a process for the 'surplus ratio'—the excess of consumption over habit, relative to consumption—rather than an evolution for the habit stock. This paper shows that Campbell–Cochrane preferences can be accommodated in a Markov chain framework à la Mehra and Prescott (1985) and mapped into an equivalent state-dependent form of the sort studied by Melino and Yang (2003). The equivalence sheds light on the workings of Campbell–Cochrane preferences and the plausibility of upcounting in Melino and Yang's framework. The result may also have some pedagogical value.

1. Introduction

In this note, I demonstrate one way of putting the habit preferences of Campbell and Cochrane (1999) into the two-state Markov chain framework of Mehra and Prescott (1985). I expect a natural question in the minds of at least a few readers is, "Why?" The answer is that by situating Campbell–Cochrane preferences in a Mehra–Prescott economy we can perform another sort of 'reverse engineering' exercise, complementary to that performed by Campbell and Cochrane themselves. The reverse engineering draws on the work of Melino and Yang (2003), who showed us, in the context of a Mehra–Prescott economy, exactly what the stochastic discount factor (SDF) must look like to match the first and second moments of asset returns in Mehra and Prescott's long sample of returns. We can calibrate our version of Campbell–Cochrane preferences to match that SDF.

The exercise would be of only pedagogical interest unless it told us something interesting about one or both of the two approaches to the equity premium puzzle that it combines. I think it does. While countercyclical risk aversion has been rightly emphasized as a key mechanism in the Campbell–Cochrane model, mapping Campbell and Cochrane into a state-dependent preference specification that matches the returns data shows that a countercyclical utility discount factor, often

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greater than one, is also important. And, while Melino and Yang dismiss state-dependent specifications that imply discount factors greater than one, the model here shows there may be a plausible story that rationalizes such a specification.

That said, the exercise may have pedagogical value as well. Given the computational tractability of the Markov chain framework, and the ubiquity of analogous structures in macroeconomics, the framework is a natural one in which to teach asset-pricing within a graduate course in macroeconomics.[1] Most of the major responses to the 'equity premium puzzle' fit easily within the framework—except for Campbell and Cochrane.[2] The exercise in this paper fills that gap.

It is useful to quickly review the features of the Campbell–Cochrane and Melino–Yang models separately before combining them. The next two sections do this.

1.1. Campbell and Cochrane

Campbell and Cochrane's 1999 paper in the *Journal of Political Economy* employs habit formation to successfully resolve a number of asset pricing puzzles, including Mehra and Prescott's equity premium puzzle. Campbell and Cochrane achieve these resolutions by a clever reverse engineering of their representative agent's habit process.

Rather than specify a law of motion for the habit stock, Campbell and Cochrane specify a law of motion for what they call the 'surplus ratio', $S_t = (c_t - h_t)/c_t$, where c_t is aggregate consumption (the habit is external) and h_t is the habit stock. Their stochastic discount factor, from t to $t+1$, is

$$m_{t,t+1} = \beta x_{t+1}^{-\alpha}\left(\frac{S_{t+1}}{S_t}\right)^{-\alpha} \tag{1}$$

where β is the utility discount factor, and the curvature parameter α, together with the surplus ratio, determines the agent's local degree of risk aversion.[3] As Campbell and Cochrane note, countercyclical risk aversion is a key feature of their specification.

Consumption growth x_{t+1} is assumed to be *i.i.d.* lognormal, and the log surplus ratio is assumed to evolve according to

$$\log(S_{t+1}) = (1-\phi)\bar{s} + \phi\log(S_t) + \lambda(S_t)[\log(x_{t+1}) - g] \tag{2}$$

where g is the mean of log consumption growth, ϕ controls the persistence of the surplus ratio process, and the crucial function $\lambda(S_t)$ controls the sensitivity of changes in the surplus ratio to shocks to consumption growth.[4]

The key to their reverse-engineering is the form of $\lambda(S_t)$,which is decreasing in S_t, hence countercyclical.

1.2. Melino and Yang

Melino and Yang, in their 2003 paper in the *Review of Economic Dynamics*, perform another type of reverse engineering exercise. Using Mehra and Prescott's two-state Markov chain for consumption growth, and assuming that consumption growth is a sufficient statistic for the riskless rate and the price-dividend ratio of an aggregate consumption claim, they derived the stochastic discount factor that, in combination with the Mehra–Prescott consumption process, yields equity and riskless return processes that exactly match the means and standard deviations calculated by Mehra and Prescott from their long sample of asset returns.

Recall that the Mehra–Prescott Markov chain has

$$x_t \in \{x_L, x_H\} = \{0.982, 1.054\} \tag{3}$$

and

$$P = \begin{bmatrix} P_{LL} & P_{LH} \\ P_{HL} & P_{HH} \end{bmatrix} = \begin{bmatrix} 0.43 & 0.57 \\ 0.57 & 0.43 \end{bmatrix} \tag{4}$$

where $P_{ij} = \Pr\{x_{t+1} = x_j : x_t = x_i\}$. Here, L and H denote the low and high consumption growth states, respectively. Mehra and Prescott's long sample of data on returns has an average risk-free rate of 0.8% and an average equity return of 7%. The standard deviations of the risk-free rate and equity return are 5.6 percentage points and 16.5 percentage points, respectively.

The Melino-Yang SDF is

$$\hat{m} = \begin{bmatrix} \hat{m}_{LL} & \hat{m}_{LH} \\ \hat{m}_{HL} & \hat{m}_{HH} \end{bmatrix} = \begin{bmatrix} 1.86 & 0.24 \\ 1.13 & 0.95 \end{bmatrix} \tag{5}$$

[1] For an example, see the 'Markov Asset Pricing' section in John Stachurski and Thomas Sargent's online lectures on quantitative economics: http://quant-econ.net/py/markov_asset.html.

[2] Models with Epstein-Zin preferences, rare disasters, concerns for robustness, disappointment aversion, and (with some effort) long-run consumption risk can all be treated computationally as simple extensions of the Mehra and Prescott's framework.

[3] Campbell and Cochrane show that, locally, relative risk aversion is given by $\left|\frac{u''(c_t-h_t)c_t}{u'(c_t-h_t)}\right| = \frac{\alpha}{S_t}$. Here and below, our notation differs slightly from Campbell and Cochrane's.

[4] Campbell and Cochrane write λ as a function of $\log(S_t)$, but that difference is immaterial here.