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Technical Note

Multiobjective topology optimization for finite periodic structures

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ABSTRACT

Many engineering structures consist of specially-fabricated identical components, thus their topology optimizations with multiobjectives are of particular importance. This paper presents a unified optimization algorithm for multifunctional 3D finite periodic structures, in which the topological sensitivities at the corresponding locations of different components are regulated to maintain the structural periodicity. To simultaneously address the stiffness and conductivity criteria, a weighted average method is employed to derive Pareto front. The examples show that the optimal objective functions could be compromised when the total number of periodic components increases. The influence of thermoelastic coupling on optimal topologies and objectives is also investigated.

1. Introduction

Engineering structures are often devised with two significant features. Firstly, most structures are expected to function in different physical conditions; and secondly, many structures comprise some identical components or parts for mass production, storage and transportation benefits. This is why the structural optimization of outstanding multifunctional objectives with periodic components is of particular importance in engineering context.

Over the last two decades, topology optimization has been developed as an effective tool to seek the optimal configuration of a structure for multidisciplinary criteria in a specified design domain [1–5], in which substantial efforts have been devoted to different algorithms, formulations and solutions to various individual criteria, ranging from mechanical [1,2,4–6] to thermal [7–9], permeable [10,11] and magnetic [12] objectives. Some attempts have been made to optimize various coupled multiphysical systems, e.g. piezoelectric [13,14], thermoelastic [15–19] and thermoelectrical [20,21] designs.

One of the main challenges confronted in topology optimization is the involvement of more than one design objective, in particular, those competing criteria [22,23]. It is often difficult to achieve all the design objectives simultaneously and certain trade-off must be made during the design. To cope with this issue, an accepted alternative is to derive a Pareto optimum, where a Pareto front is generated and the multiple objectives are optimized in a compromise manner. In this respect, the linear weighting function scheme (i.e. arithmetic average) has been extensively applied to search for multiobjective optimum provided that the Pareto front is convex. It is noted that the thermoelastic problems have been often exemplified to demonstrate such design features. Li et al. [24] and Kim et al. [25] adopted the weighting factor method to combine thermal stress and heat flux for a unified design criterion, and showed that varying the weights led to different topologies. de Kruijf et al. [26] incorporated the stiffness and conduction criteria into a single cost objective for 2D structural and material designs. However, a more thorough study is still needed, particularly to explore the relationship between multiobjective optimal topologies and corresponding Pareto fronts.

Another key issue that remains to be under-investigated is the topology optimization for periodic structures that comprise a finite number of identical components or parts. Unlike the periodic materials whose base cells are generally many orders smaller than the materials sample considered [27,28], the sizes of periodic components are often comparable to the entire structural system. In this scenario, external boundary and loading conditions could significantly affect each component to different extents, making their performances inhomogeneous [29]. Thus the typical homogenization technique [30] may not be applicable for relating the global properties to base cell (component) characteristics. In other words, the inverse homogenization algorithms [27] developed for periodic material design may be inapplicable. To tackle this problem, Zhang and Sun [31] developed a two-level design approach by combining the macroscopic optimization with microscopic optimization. More recently, Huang and Xie [32] established a mono-scale approach to the optimization of periodic structures, by simply averaging the sensitivities at corresponding locations of each component to maintain the structural periodicity. In these two articles, however, only a simple stiffness criterion has been considered.

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This paper aims to develop a unified procedure for 3D multiobjective topology optimization of finite periodic structures. Without loss of generality, the minimizations of mechanical and thermal compliances are adopted as the design objectives herein. A number of examples are presented to explore the topologies, Pareto fronts and assembly patterns.

2. Statement of the problem and sensitivity analysis

The finite element solutions to elastic and thermal governing equations determine the displacement field $\mathbf{u}$ and temperature field $\boldsymbol{\phi}$, which allow us to assess the mechanical and thermal performances of a specific structure in terms of the compliances. It is deemed that the lower the mechanical and thermal compliances, the better the corresponding performances. In a multiobjective framework, the design problem can be accordingly formulated in terms of weighting factors w_s and w_c, as

$$\begin{cases} \min F &= w_s f_s + w_c f_c \\ &= \sum_{e=1}^{NE}\left[w_s\dfrac{(\mathbf{u}_s^e+\tau\mathbf{u}_c^e)^T(\rho^e)^{p_s}\mathbf{K}_s^e(\mathbf{u}_s^e+\tau\mathbf{u}_c^e)}{C_s^*}+w_c\dfrac{(\boldsymbol{\phi}^e)^T(\rho^e)^{p_c}\mathbf{K}_c^e(\boldsymbol{\phi}^e)}{C_c^*}\right] \\ \text{s.t.} & \sum_{e=1}^{NE}V^e\rho^e=V_t;\quad 0\leqslant\rho_{\min}\leqslant\rho^e\leqslant 1,\quad \tau=0,1; \\ & e=1,2,\ldots,NE \end{cases} \tag{1}$$

where f_s and f_c are mechanical and thermal compliance objectives, respectively. NE denotes the total number of elements, V_t the volume constraint and ρ_{min} represents the lower limit of design variable ρ^e (relative density), preventing finite element analysis from singularity [33–35]. u_s^e, u_c^e and ϕ^e are the elemental displacement vectors due to the mechanical and thermal loadings and elemental temperature vector, respectively. K_s^e and K_c^e are the elemental stiffness and conductivity matrices. C_s^* and C_c^* are the maximum structural and thermal compliances which are used to normalize these two different objective functions to a range of 0–1. p_s and p_c are two parameters used in the SIMP model to penalize intermediate densities towards a 0–1 design [1]. w_s and w_c, which are chosen such that the sum equals unity, i.e. $w_s + w_c = 1$, represent two weighting factors to control the proportion or emphasis between mechanical and thermal objectives.

The sensitivity of ith element for the bi-objective optimization problem defined in Eq. (1) can be given as,

$$\frac{\partial F}{\partial\rho^i} = -w_s p_s\frac{\left(\mathbf{u}_s^i+\tau\mathbf{u}_c^i\right)^T(\rho^i)^{(p_s-1)}\mathbf{K}_s^i\left(\mathbf{u}_s^i+\tau\mathbf{u}_c^i\right)}{C_s^*} - w_c p_c\frac{(\boldsymbol{\phi}^i)^T(\rho^i)^{(p_c-1)}\mathbf{K}_c^i(\boldsymbol{\phi}^i)}{C_c^*} \tag{2}$$

It should be noted that in the finite periodic structure, the difference of two physical fields in different components make the sensitivities non-periodic, thereby leading to a non-periodicity of different components after optimization. To avoid such a paradox, Huang and Xie [32] proposed a simple yet effective method, in which all the sensitivities at the corresponding locations of different components are averaged as

$$\frac{\partial F}{\partial\rho^{i,j}} = -\frac{1}{NP}\sum_{j=1}^{NP}\left[\begin{array}{l} w_s p_s\dfrac{\left(\mathbf{u}_s^{i,j}+\tau\mathbf{u}_c^{i,j}\right)^T(\rho^{i,j})^{(p_s-1)}\mathbf{K}_s^{i,j}\left(\mathbf{u}_s^{i,j}+\tau\mathbf{u}_c^{i,j}\right)}{C_s^*} \\ +w_c p_c\dfrac{(\boldsymbol{\phi}^{i,j})^T(\rho^{i,j})^{(p_c-1)}\mathbf{K}_c^{i,j}(\boldsymbol{\phi}^{i,j})}{C_c^*} \end{array}\right] \tag{3}$$

where $(\cdot)^{i,j}$ denotes the ith element in the jth component and $NP = n_x \times n_y \times n_z$ is the total number of components. By doing so, a unified sensitivity of different components is provided, which allows us to optimize the topologies of different components in a consistent way. Consequently, the structural periodicity can be retained via such a variable-linking technique as indicated in Eq. (3), in which the elemental densities at corresponding locations are preserved to have the same value, thereby maintaining the periodicity during the design.

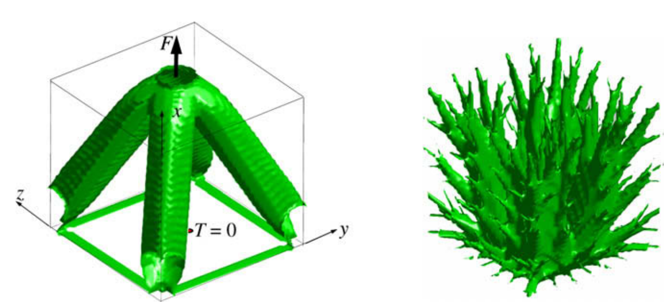
3. Results and discussion

In this section, we will present three demonstrative examples to illustrate the bi-objective designs with or without coupling effect for both conventional ($1 \times 1 \times 1$) and finite periodic topology optimizations. All the 3D design domains are discretized into unit cubic elements and the initial density fields are of uniform material distribution where volume fraction is equal to the prescribed one. In this paper, the penalty factors used in the SIMP models for the Young's modulus and conductivity are both set as $p_s = p_c = 3$ [36]. During the optimization process, it will not be considered convergent until the maximum density change in any element is less than 0.1% in 10 consecutive iterations.

3.1. Conventional structural designs with bi-objectives

As shown in Fig. 1(a), the design domain is evenly heated at all nodes in this example. At the center on the bottom surface, there is a heat sink, whose temperature is kept to be zero degree. Four corners of the bottom surface are kinematically fixed and a unit force is applied along the x-axis at the center of the upper surface. Due to the double-symmetry of the structure, we only analyze a quarter of the design domain that is discretized into $80 \times 40 \times 40$ unit cubic elements. The Young's modulus, Poisson's ratio and the conductivity of solid materials are set to 1, 0.3 and 1, and the constraint of volume fraction is 30%, respectively.

By varying w_s from 1 (the full stiffness design) to 0 (the full conduction design), the optimal structural topologies are generated as shown in Fig. 1. It is interesting to see how the mechanical and thermal objectives compete with each other and make significant influence on the topological designs. In the full stiffness design ($w_s = 1$, Fig. 1(a)), the four bars connect the fixed corner boundary with the loading point to best support the external mechanical force. On the other hand, in the full conduction design ($w_s = 0$, Fig. 1(b)), the material is mostly distributed near the heat sink and spread out along a doubly-symmetric tree-like configuration with numerous fine twigs. In this case, there is no any connection between the kinematic boundary (the four bottom corners) and the mechanical loading point. However, if slightly increase the stiffness weight (e.g. $w_s = 0.001$), an evident connection near the load point can be observed as shown in Fig. 2(a). Further increase in the stiffness weight will strengthen the connection of loading point to kinematic boundary and weaken the heat dissipation from the heat



(a) $w_s = 1.000$ (b) $w_s = 0.000$

Fig. 1. Two extreme cases in conventional bi-objective structural designs.