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Propagation of mechanical waves through a stochastic medium with spherical symmetry

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HIGHLIGHTS

- An extended spherical Bessel differential equation governs the average displacement.
- Additional terms in differential equation contain the stochastic contribution.
- Waveform amplitudes depend on the autocorrelation length and the noise intensity.
- Autocorrelation length variations predominate over fluctuation intensity changes.

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ABSTRACT

We theoretically analyze the propagation of outgoing mechanical waves through an infinite isotropic elastic medium possessing spherical symmetry whose Lamé coefficients and density are spatial random functions characterized by well-defined statistical parameters. We derive the differential equation that governs the average displacement for a system whose properties depend on the radial coordinate. We show that such an equation is an extended version of the well-known Bessel differential equation whose perturbative additional terms contain coefficients that depend directly on the squared noise intensities and the autocorrelation lengths in an exponential decay fashion. We numerically solve the second order differential equation for several values of noise intensities and autocorrelation lengths and compare the corresponding displacement profiles with that of the exact analytic solution for the case of absent inhomogeneities.

1. Introduction

Analysis concerning propagation of waves in stochastic media stems from the fact that in most of real media are present inhomogeneities and indeterminacies in their structure. The basic phenomenon in stochastic propagation is the wave scattering, this is, scattered waves are superimposed on the primary incident wave and cause fluctuations of the characteristics of the total field including attenuation of the incident wave [1].

Propagation of waves in random media has been widely studied [2–11] among many others and recent research remains ongoing [12–16]. Examples of well-established models for propagation within polycrystalline materials are the Stanke & Kino [6] and Weaver-type [10] second-order models (SOMs). The Weaver model, on which the SOM is based, is an extension of the Bethe–Salpeter (mean intensity) and the Dyson (for the mean field) [10,17] formalisms found in electromagnetics. The

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mentioned formalisms are useful to study propagation and scattering in terms of the perturbed complex wave propagation constant denoting the scattering-induced attenuation and velocity, dispersion characteristics for a spatially coherent plane wave. This approach usually consists in obtaining the scattering by lower orders, such as single scattering [11], restricting their validity to weakly scattering situations. Multiple scattering theories [18] are restrained by the energy transfer equations which govern the propagation and represent it as a diffuse wave phenomenon.

Recently, large scale random simulations are being assessed in various fields using elaborated numerical approaches. Among these algorithms, the stochastic collocation and the stochastic Galerkin procedure are widely utilized. Stochastic collocation formalisms [19–23] can be understood as a Monte Carlo type sampling algorithm besides the fact that, instead of a random point, the sampling points are chosen following a certain kind of numerical quadrature schemes which are employed to evaluate the statistical moments of the solution. Additionally, in the field of solid mechanics, stochastic Galerkin approaches are employed mainly to solve static problems with aleatory parameters [24–26]. Solutions of dynamic problems have been also considered, but mostly in the frequency domain [27], thereby restricted to linear regime. Kundu and Adhikari [28], very recently, have presented a time domain formulation of a stochastic Galerkin scheme for application in the field of structural dynamics.

In Refs. [29–32], elastic wave propagation in random media was analyzed by using the mean wave formalism where the problem is treated by a statistical method. Particularly, in [29,30] the propagation of a wave in a random medium is analyzed on the assumption that the medium differs slightly from a homogeneous medium and an integro-differential equation satisfied by the average wave is deduced which is correct up to terms of second order in a parameter characterizing the deviation of the medium from homogeneity. The authors apply their general theory to the elastic wave propagation in a medium in which the Lamé coefficients and density are all random functions of position and they found that the propagation constant for the average or coherent wave is complex even for a nondissipative medium, because the coherent wave is continually scattered by the inhomogeneities and converted into the incoherent wave. In Ref. [29], Keller proposed the name of "honest method" to his statistical general theory. According to his proposal, "dishonest methods" are those in which the average of a product is replaced by a product of averages. An example of this approach can be found in [33], where Bourret obtained an integro-differential equation for the ensemble mean value of a field by treating the perturbed field and quadratic terms in the perturbing field as statistically independent. This hypothesis of local independence amounts to regarding the stochastic field as having a negligible local effect on the perturbed field. In Refs. [34,35], van Kampen solved a general stochastic linear differential equation with an initial condition by a systematic expansion for the expectation value of the process and the author compared it with Bourret and Keller methods. If α is a parameter measuring the magnitude of the fluctuation and τ_c is the finite autocorrelation time, the differences between the Bourret and van Kampen approaches are negligible up to order $(\alpha\tau_c)^2$. Although Keller criticized "dishonest methods", under certain circumstances, these methods have been applied successfully in several papers, for example in [36–38], for $\alpha\tau_c \ll 1$.

A primary problem of seismology is the determination of displacements in the interior of an elastic solid due to an impulsive compressional source located in the inner of the medium. Deterministic mathematical description of homogeneous systems with sources generating spherically diverging elastic waves can be found in the literature [39–42]. As said above, real media present random inhomogeneities which must be taken into account in studying elastic wave propagation. Under these circumstances, a stochastic approach based on a small number of statistical parameters characterizing the inhomogeneities is preferred [43]. It is shown that the wave envelope-broadening phenomenon observed in seismographs can be better explained by modeling envelope of spherically outgoing waves radiated from a point source [44,45]. For the propagation of seismic waves radiated from a microearthquake in the lithosphere, the authors of Ref. [44] developed a direct simulation method of spherically outgoing scalar waves radiated from a point source in a random media characterized by a von Karman-type autocorrelation function. Later, in 2007, Sato [45] studied the case when the weak random velocity inhomogeneity is characterized by a Gaussian autocorrelation function and the wavelength is shorter than the correlation distance.

In Refs. [43–45] the elastic waveforms have been treated as a sum of unperturbed waves satisfying the wave equation for a homogeneous medium and scattered waves by distributed inhomogeneities. Instead of previous treatment, in our work, considering an ensemble of random media, we will evaluate physical quantities by taking the average over the ensemble of random media. Our research is focused on the study of the propagation of scalar outgoing mechanical waves through an infinite isotropic elastic medium with spherical symmetry whose Lamé coefficients and density are spatial random functions characterized by an Ornstein–Uhlenbeck autocorrelation function considering that they have a simple exponential decay. Based on the method developed in Ref. [34], we derive a linear differential equation for the expectation value of the displacement as function of a parameter measuring the magnitude of the fluctuation and the finite autocorrelation length. Even though the method of solution proposed by Keller can be used for solving systems possessing spherical symmetry, his method involves the solution of an integro-differential equation which requires an integration over previous values of displacement. Mathematically, Keller's method is essentially more complicated than the process of solving a linear differential equation proposed by van Kampen. Our results clearly exhibit the usefulness of van Kampen's approach in solving wave propagation in elastic media. The outline of our paper is as follows. Section 2, contains the basic deterministic equations governing the radial propagation of elastic waves through an isotropic and inhomogeneous medium possessing complete spherical symmetry. Next, in Section 3 we apply van Kampen's method to the obtained equation in previous section by considering that the medium properties have small fluctuations around the deterministic values. Section 4 shows our results and their respective analyses. Finally, the conclusions are presented.