Physica A 490 (2018) 613–623

Contents lists available at ScienceDirect

Physica A

journal homepage: www.elsevier.com/locate/physa

Dynamic analysis of a stochastic rumor propagation model

Fangju Jia [a], Guangying Lv [a,b,*]

[a] School of Mathematics and Statistics, Henan University, Kaifeng, Henan 475001, China
[b] Institute of Mathematics, School of Mathematical Science, Nanjing Normal University, Nanjing 210023, China

HIGHLIGHTS

- A new stochastic rumor propagation model is studied.
- Sufficient conditions for extinction and persistence in the mean are given.
- The threshold, which is smaller than that of deterministic system, between persistence in the mean and extinction is obtained.

ARTICLE INFO

Article history:
Received 5 April 2017
Received in revised form 29 May 2017

MSC:
34F05
60H10
93E03

Keywords:
Rumor propagation model
Itô's formula
Threshold

ABSTRACT

The rapid development of the Internet, especially the emergence of the social networks, leads rumor propagation into a new media era. In this paper, we are concerned with a stochastic rumor propagation model. Sufficient conditions for extinction and persistence in the mean of the rumor are established. The threshold between persistence in the mean and extinction of the rumor is obtained. Compared with the corresponding deterministic model, the threshold affected by the white noise is smaller than the basic reproduction number R_0 of the deterministic system.

1. Introduction

With the rapid development and wide popularity of the Internet, the way to acquire information for people is being quietly changed. The new information platforms appear, and social networks, as a typical example, have been widely recognized and applied due to its flexibility, convenience, openness and low cost. For example, the media commonly releases their latest information through social networks. Social networks can bring convenience and efficiency to daily life and information exchange, but it also results in gradual prevalence of online rumor. Especially, in recent years, with the rise of the mobile communications equipment, such as smart-phones and tablet computers, social network rumors have shown a surge in trend.

Rumor was regarded as collective problem-solving, see [1,2], in which people caught in ambiguous situations try to construe a meaningful interpretation . . . by pooling their intellectual resources. It is remarked that social network rumors usually include the following characteristics: fabricating false news, seeking economic interests, aggression, retaliation, abreaction and so on. Recently, social network rumor propagation is increasingly becoming a hot research direction.

* Corresponding author at: School of Mathematics and Statistics, Henan University, Kaifeng, Henan 475001, China.
E-mail address: gylvmaths@henu.edu.cn (G. Lv).

http://dx.doi.org/10.1016/j.physa.2017.08.125
0378-4371/

Particularly, mathematical modeling has played an increasingly important tool in describing rumor diffusion in social networks. Zhao–Zhu [3] introduced the following model

$$\begin{cases} \dfrac{\partial S}{\partial t} = d\dfrac{\partial^2 S}{\partial x^2} + A - \beta SI - \mu S + \alpha I^2 \\ \dfrac{\partial I}{\partial t} = d\dfrac{\partial^2 I}{\partial x^2} + \beta SI - (\mu+\eta)I - \alpha I^2, \end{cases}$$

where $t > 0, x \in D = (0, L)$ with homogeneous Neumann boundary conditions

$$\frac{\partial S}{\partial \nu}(t,x) = \frac{\partial I}{\partial \nu}(t,x) = 0, \quad t \geq 0, \quad x \in \partial D,$$

and

$$\begin{cases} S(0,x) = \rho_1(x), \quad x \in \bar{D}, \\ I(0,x) = \rho_2(x), \quad x \in \bar{D}, \end{cases}$$

where $S(t, x)$ and $I(t, x)$ represent the densities of the rumor susceptible users and the rumor infected users with a distance of x at time t, respectively. $d\frac{\partial^2}{\partial x^2}$ is a diffusion term, being used to describe the impact of the mobility on the rate of change in the density of users with a distance of x at time t. L describes the upper bound of the distances between the rumor and other social network users. We only recall the meaning of α: when a rumor infected user contacts another rumor infected user, one rumor infected user will further analyze the other infected users' opinions about the rumors and make an accurate judgment, then the rumor infected user may transform into a rumor susceptible user at rate α. Therefore, the rumor propagation model is different from the usual epidemic model. About the meaning of positive parameters A, β, μ, η and α, see [3] for more details.

Let us recall the history of rumor propagation model. The research on rumor propagation models started in the 1960s. Daley–Kendall [4] first divided people into three classes: ignorant, spreaders and stiflers, and then developed a stochastic rumor propagation model (D–K model) based on the infectious disease research method. Maki–Thompson [5] believed that rumors were disseminated through the two-way contact between the disseminators and other people in the crowd. By modifying D–K model, they introduced a Maki–Thompson model to describe rumor spreading based on Markov chain. Zanette [6] first studied the dynamic behavior of rumor propagation in small world networks, and similarly to disease models, he obtained the spreading threshold. Recently, based on the classical Susceptible–Infected–Removed rumor propagation model, Zhao et al. [7] developed a new Susceptible–Infected–Hibernator–Removed model by adding a direct link from ignorants to stiflers and a new group hibernator. Afassinou [8], through comparing the behavior of educated individuals and non-educated individuals, analyzed the impact of education rate on the rumor spreading mechanism, also see [9].

It is easy to see that the above rumor model is similar to most of the infectious disease models, and there are some difference between the rumor model and epidemic model, see [3] for more details. Thus it worth studying it.

For simplicity, in this paper, we do not consider the spatial effect, that is to say, $d = 0$. On the other hand, in the natural world, rumor model are always affected by the environmental noise, which is similar to epidemic model, see [10–16]. It is necessary to reveal how the environmental noise affects the rumor model. The stochastic models, which is usually described by stochastic differential equations, can predict the future dynamics of the system.

Following the idea of Jiang et al. [11], in this paper, we suppose that stochastic perturbations are of the white noise type which are directly proportional to $S(t)$ and $I(t)$, influenced on the $\dot{S}(t)$ and $\dot{I}(t)$ in the rumor model. Hence, the stochastic rumor model will take the following form

$$\begin{cases} dS(t) = [A - \beta S(t)I(t) - \mu S(t) + \alpha I^2(t)]dt + \sigma_1 S(t)dB_1(t) \\ dI(t) = [\beta S(t)I(t) - (\mu+\eta)I(t) - \alpha I^2(t)]dt + \sigma_2 I(t)dB_2(t), \end{cases} \tag{1.1}$$

with initial data

$$\begin{cases} S(0) = s_0, \\ I(0) = i_0, \end{cases}$$

where A, β, μ, η and α are all positive constants, and $B_i(t)$ are mutually independent standard Brownian motions with $B_i(0) = 0, \sigma_i^2 > 0$ denote the intensities of the white noise, $i = 1, 2$.

The main purpose of this paper is to investigate the asymptotic properties of system (1.1). This paper is organized as follows. In Section 2, we prove that there exists a unique global positive solution of (1.1). Section 3 is concerned with the extinction of the rumor, that is, we will establish sufficient condition for the rumor to disappear. The condition for the rumor being persistent is obtained in Section 4.

Throughout this paper, we let $(\Omega, \mathcal{F}, \mathbb{P})$ be a complete probability space with a filtration $\{\mathcal{F}_t\}_{t\geq 0}$ satisfying the usual condition. In addition, $B_i(t), i = 1, 2$ are defined on the probability space $(\Omega, \mathcal{F}, \mathbb{P})$.