Computers and Structures 165 (2016) 76–95

Contents lists available at ScienceDirect

Computers and Structures

journal homepage: www.elsevier.com/locate/compstruc

Modeling of nonlocal damage-plasticity in beams using isogeometric analysis

Guodong Zhang [a], Kapil Khandelwal [b,*]

[a] Dept. of Civil & Env. Engg. & Earth Sci., University of Notre Dame, United States
[b] Dept. of Civil & Env. Engg. & Earth Sci., 156 Fitzpatrick Hall, University of Notre Dame, Notre Dame, IN 46556, United States

ARTICLE INFO

Article history:
Received 17 August 2015
Accepted 16 December 2015
Available online 7 January 2016

Keywords:
Nonlocal damage
Resultant plasticity
Isogeometric analysis
Beam finite elements
Implicit gradient approximations

ABSTRACT

A nonlocal damage-plastic Euler–Bernoulli beam model is proposed based on resultant damage-plasticity together with implicit approximations of the nonlocal integral operators. Isogeometric concept is used to approximate the higher order gradient terms, and numerical algorithms and consistent tangents are derived. Test cases are presented to show the important features of the proposed nonlocal beam element. Numerical results show that the proposed nonlocal beam is successfully able to preserve the objectivity of the results and a meaningful convergence is achieved with decreasing mesh size. In addition, various softening responses associated with localized damage mechanisms – including snap-back and snap-through responses – are obtained.

1. Introduction

Successful simulation of structural system response under extreme events requires appropriate models that incorporate the physics of associated damage and failure with high fidelity. To this end, macromodels that utilize a combination of beams and discrete finite elements are well suited for the overall global response simulation [1–3]. A key constituent of these macromodels is the description of inelastic zones wherein energy dissipation occurs and the associated length-scale effects [4–7], and appropriate models of dissipative damage mechanisms are required to obtain physically reasonable results. In the case of beams, damage due to dissipative mechanisms is typically confined to critical locations termed "inelastic-hinge" regions, and the response is usually accompanied by localization of damage at these locations [8,9]. When undergoing damage localization, significant internal force redistribution may occur under extreme loads. Therefore, it is imperative to simulate with high fidelity the physics of localized damage mechanisms to capture the associated internal force redistribution and the resulting system behavior.

In the past, various inelastic hinge models have been proposed for capturing the plastic response and the associated damage and softening behavior in beam elements. Examples include lumped models [8,10,11], distributed models [12–14], and nonlocal models [15–17], among others. The introduction of softening behavior in inelastic hinges, which is necessary to model damage and capture the load redistribution, make these models length-scale dependent, and further regularization is required to obtain results that are finite element mesh size independent [6]. For instance, in lumped hinge models, the constitutive behavior of an inelastic hinge is tied to a fixed hinge length [10,11]; however, a main drawback of this approach is that the plasticity and damage can only evolve at the hinge locations that have to be specified a priori. In the distributed hinge models, on the other hand, the inelasticity can occur at any location in the beam, and in this case, regularization is typically carried out by making the constitutive response dependent on characteristic mesh size [2,12]. Another approach for addressing the softening effect is to consider nonlocal behavior with the inclusion of nonlocal terms in the constitutive models [15–18]. Moreover, two different methodologies are typically employed in such nonlocal theories [19–21]: (a) Integral based nonlocal theories – in this case, nonlocal variables are calculated as the weighted average value of the corresponding local variables and depends on a prescribed length-scale. Although an attractive approach from a theoretical point of view, these nonlocal constitutive models are difficult to implement and are numerically inefficient [22]; (b) Gradient based nonlocal theories – in this case, implicit or explicit gradient approximations of the nonlocal integral operator are used to model the nonlocal effects. Most of the studies on gradient-based models have focused on second-order gradient approximations, since only C^0-continuity of the finite element interpolation functions is required in this case. However, other studies have shown that the inclusion of higher order

* Corresponding author.
E-mail address: kapil.khandelwal@nd.edu (K. Khandelwal).

http://dx.doi.org/10.1016/j.compstruc.2015.12.006
0045-7949/

gradient terms is necessary to approximate the integral nonlocal operators [22,23]. Furthermore, finite element interpolation functions of higher order continuity are needed to meet the increased continuity requirements associated with higher order gradient approximations.

In this study, a displacement based nonlocal damage-plastic Euler–Bernoulli beam model is proposed for response simulation of nonlinear beams and frames that can be used for modeling of plasticity and the associated nonlocal damage resulting in a softening response. A distributed resultant nonlocal damage-plasticity model is introduced following the work of Engelen et al. [20] and Hjelmstad and Taciroglu [14], together with implicit approximations of the nonlocal gradient operators. Finite elements are formulated using the isogeometric concept introduced by Hughes et al. [24] and the consistent tangent modulus for the resulting nonlocal beam element is derived. Isogeometric analysis (IGA) uses B-splines for interpolation of the geometry and the solution spaces, and offers an alternative methodology for constructing higher order interpolation functions that can be effectively used for approximation of the gradient terms in the implicit nonlocal approximations. IGA offers many advantages over conventional finite element bases including exact representation of geometry, higher continuity of the basis functions and variation diminishing properties [24,25], among others. Since the beam has relatively simple geometry, the IGA's advantage of preserving the exact geometry is less important in this case. However, the higher order continuity and smoothness of the B-spline interpolations is exploited for constructing smooth finite element interpolation functions for nonlocal beam elements. Various test cases are presented to show the application and the important features of the proposed nonlocal inelastic beam element. The paper is organized as follows: Section 2 provides the theoretical description of the proposed nonlocal Euler–Bernoulli beam. Isogeometric analysis is reviewed in Section 3, and the isogeometric finite element beam formulation is presented in Section 4. Various numerical examples including convergence studies, comparison of different order of gradient models, parametric studies and length-scale effects are shown in Section 5. Finally, the important conclusions of this study are given in Section 6.

2. Nonlocal Euler–Bernoulli beam

2.1. Kinematics and kinetics

A planar geometrically nonlinear Euler–Bernoulli beam is considered with small-strain moderate rotation assumption as shown in Fig. 1. The displacement field, $\boldsymbol{u}(x) = \{u(x), v(x)\}$, comprises of the horizontal displacement $(u(x))$ and the vertical displacement $(v(x))$ of the points along the centroidal axis of the beam. In Fig. 1, $q(x)$ and $\tau(x)$ are the vertical and axial traction per unit length, and $P(x)$, $V(x)$ and $M(x)$ are the resultant axial force, shear force and bending moment at a cross-section, respectively. With the assumption of moderate rotations, the strain is expressed as

$$\varepsilon_{mr}(x) = \varepsilon_a(x) - y\kappa(x) \tag{1}$$

with

$$\varepsilon_a(x) = \frac{du}{dx} + \frac{1}{2}\left(\frac{dv}{dx}\right)^2 \tag{2}$$

$$\kappa(x) = \frac{d^2 v}{dx^2} \tag{3}$$

where $\varepsilon_a(x)$ is the axial strain that includes the nonlinear moderate rotation term $\frac{1}{2}\left(\frac{dv}{dx}\right)^2$ and $\kappa(x)$ is the curvature. Equilibrium equations for the Euler–Bernoulli beam are expressed as [14]

$$\frac{dP}{dx} = -\tau(x) \tag{4}$$

$$\frac{d^2M}{dx^2} - \frac{d}{dx}\left(P\frac{dv}{dx}\right) = q(x) \tag{5}$$

2.2. Constitutive model

A nonlocal resultant damage-plasticity model is proposed to describe the loss of strength and stiffness due to damage during an inelastic process under the applied loads. To this end, the strain

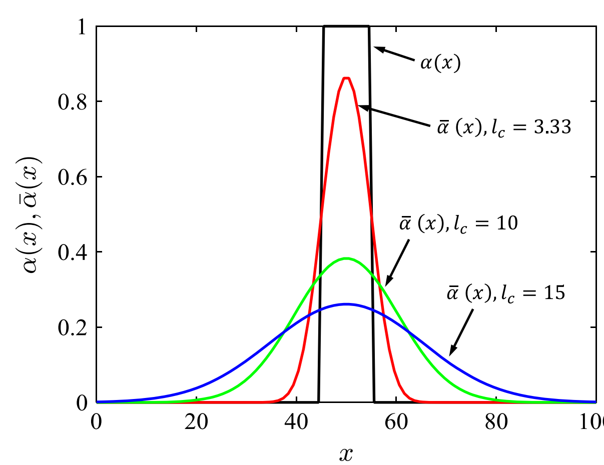

Fig. 2. Nonlocal regularization $(\bar{\alpha}(x))$ of a local field $(\alpha(x))$ for different length scales $(\Omega = [0, 100])$.

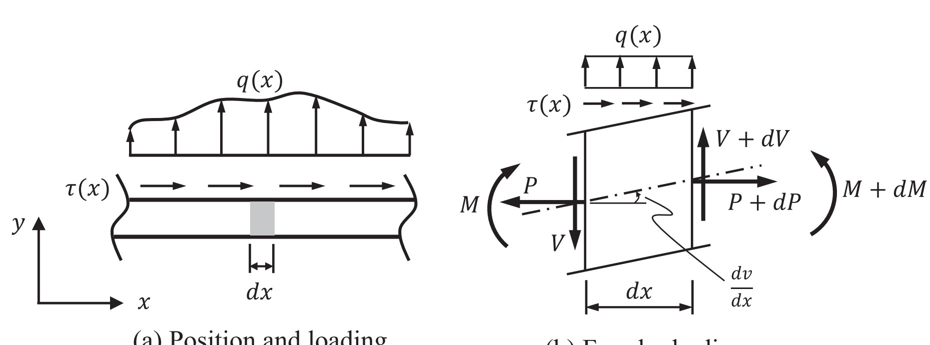

Fig. 1. Planar Euler–Bernoulli beam.