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Hierarchical design of structures and multiphase material cells

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ABSTRACT

This paper presents a hierarchical topology optimization method to simultaneously achieve the optimum structures and multiphase material cells for minimum system thermal compliance. Macro design variables and micro phase design variables are introduced independently, and coupled through elemental phase relative density. Based on uniform interpolation scheme with multiple materials, the sensitivities of thermal compliance with respect to the design variables on the two scales are derived. Correspondingly, the hierarchical optimization model of structures and multiphase material cells is built under prescribed volume fraction and mass constraints. The proposed method and computational model are validated by several 2D numerical examples. The superiority of multiphase materials in hierarchical optimization is presented through the comparison of single phase materials. The optimized results of periodic structure, hierarchical structure and traditional continuous structure are compared and analyzed. At last, the effects of volume fraction and mass constraints are discussed.

1. Introduction

Structural optimization is drawing more attention than ever before with the shortage of global resources and higher levels of world-wide competition in industries. Comparing shape optimization and size optimization, topology optimization is considered more efficient in decreasing weight at the conceptual design stage. Currently, topology optimization technology has been widely applied in various fields since it was proposed by Bendsoe and Kikuchi [1] in 1988. Periodic porous material like cellular materials and truss-like materials are always of high interest in automobile, aerospace and other industrial applications for exhibiting properties such as a high ratio of stiffness to weight, excellent energy absorption and thermal isolation characteristics. In recent years, the topology optimization method has been a powerful tool to design the multifunctional material microstructures [2–7] since inverse homogenization method was proposed by Sigmund [8]. In the past, the topology optimization method was mainly used to solve single scale optimization problems either for the optimal design of macrostructures to improve structural performance or for the material design to develop new microstructures with prescribed or extreme properties. However, the material microstructures with certain equivalent properties are not always guaranteed to be efficient when constructing structures, since both structural shapes and boundary conditions always vary in practical use. That is, we need a kind of system-level optimization technology which can embody the structural performance and material properties together. Although macro structural optimization and material microstructure design are at two different scales, they have a common feature, which is that they both focus on material distribution. The common feature supplies a possibility to integrate macrostructure and material microstructure into one system, which can offer its own set of strengths. To some extent, the integrated optimization can be understood as a material microstructure design method which can satisfy macro structure performance. Rodrigues et al. [9] proposed a hierarchical optimization method of structure and porous material, and Coelho et al. [10] extended this hierarchical approach in 3D elastic structures. However, this work strongly aims to achieve optimal material microstructures and allows variation from point to point in macro-scale, which leads to low computational efficiency and some difficulties in manufacture. A universal approach for concurrent design at two scales was proposed by Liu et al. [11]. Material is assumed to be uniformly distributed on the macro level. Penalization approaches are adopted at two scales to achieve clear topologies, i.e. porous anisotropic material penalization at the macro-scale, and conventional SIMP at the micro-scale. Optimizations at two scales are integrated into one system through homogenization theory. This method is recommended for easier manufacturing, although optimal design may not be achieved. Similar optimization models were applied by Yan et al. [12] and

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Niu et al. [13] to account for thermo-mechanical loads and frequency optimization. Recently, Huang et al. [14] developed the BESO to the concurrent design of macrostructures and material microstructures, which demonstrated certain advantages over the continuum density-based method [15–18]. Zhang and Sun [19] and Yan et al. [20] investigated the size-effects of material microstructure on the concurrent optimization based on superelement technique and extended multiscale finite element method, respectively. Through numerical examples, Liu et al. [21] pointed out that structure design is oriented to structure efficiency, material design is oriented to multifunctional properties and hierarchical design is oriented to both structure efficiency and multifunctional properties. Deng et al. [22] and Yan et al. [23] indicated that, sometimes the two-scale optimization of structure and porous material is more advantageous than the sole macro structural optimization in terms of improving multi-objective performances.

Multiphase materials have been researched widely because of their advantages in complicated external conditions. A literature study points out that structural topology optimization with multiphase materials stemmed from Thomsen [24]. Sigmund and his co-worker [3,25] expanded the SIMP to interpolate material properties of two solid material phases and void. Up to now, multiphase materials have been studied by various methods [26–28]. In particular, Gao and Zhang [29] testified that the mass constraint is more effective than the volume constraint in the topology optimization of structures consisting of multiphase materials. Similar to Jacobi and Gauss–Seidel iteration processes, Tavakoli and Mohseni [30] split multiphase material problems into a series of binary material iterative problems. Derived from multiphase material topology optimization, a method labeled as Discrete Material Optimization (DMO) was proposed by Stegmann and Lund [31] to treat the laminate design of composite materials.

Different with literature [15,18], the proposed hierarchical optimization approach is to find optimum thermal conductive configurations for structures and porous multiphase material cells under prescribed volume fraction and mass constraints. There is only one finite element model. Structure and material cell are coupled through elemental phase relative density instead of homogenization theory. A uniform interpolation scheme is used to deal with multiple material problems. The layout of the paper is as follows. A hierarchical optimization model for thermal conduction is established and described in Section 2. The two-scale sensitivity analysis based on finite element method is presented in Section 3. In Section 4, the numerical treatments are given. In Section 5, several 2D numerical examples are presented to demonstrate the effectiveness of the proposed optimization algorithm. In Section 6, a summary and all drawn conclusions are provided.

2. Hierarchical topology optimization model with multiphase materials

In this research, it is assumed that structure is assembled by uniform multiphase material cells. The optimization at two scales is integrated into one system and resolved simultaneously. As shown in Fig. 1, the 2D designable domain is divided into $M \times N$ finite elements, where M stands for the total number of unit cells and N stands for the total number of finite elements within each cell. To ensure the structural periodicity, all unit cells should be meshed consistently. Two classes of design variables are independently defined, i.e. macro design variable P_i $(i = 1, 2, \dots, M)$ in structural design domain and micro phase design variable $r_j^{(q)}$ $(j = 1, 2, \dots, N,\ q = 1, 2, \dots, S$, where S stands for the varieties of solid materials) in a unit cell, both ranging from 0 to 1. $P_i = 1$ if unit cell i is occupied by multiphase material cell. $P_i = 0$ if no multiphase material cell exists in unit cell i. $r_j^{(q)} = 1$ if element j is full of phase material q. $r_j^{(q)} = 0$ if phase material q doesn't exist in element j within the material cell.

In every finite element, micro phase design variables should satisfy

$$\sum_{q=1}^{S} r_j^{(q)} = 1 \tag{1}$$

Eq. (1) reflects the mutex relationship between $r_j^{(q)}$ $(q = 1, 2, \dots, S)$, i.e. if $r_j^{(q)} = 1$, $r_j^{(\xi)} = 0$ $(\xi \neq q)$. Eq. (1) cannot be used directly in the optimization process because huge combinatorial problem is involved. Here a uniform interpolation model is used to solve the problem of multiphase materials which will be introduced in Section 3.

For a discretized structure, each element is assigned S relative densities and can be expressed as the combination of two-scale design variables [21]

$$x_{ij}^{(q)} = P_i r_j^{(q)} \tag{2}$$

In Eq. (2), i and j indicate the unit cell number and the elemental number within the unit cell, respectively. Elemental phase relative density $x_{ij}^{(q)}$ imposes the design variables P_i and $r_j^{(q)}$ associated with each element in designable domain. From Eq. (2) we know that the status of any element is identical to its corresponding ones in other existent unit cells, which guarantees the uniformity of porous material cell at a macro-scale.

To seek the minimum thermal compliance for structure and porous cell with multiphase materials, the hierarchical topology optimization can be formulated as

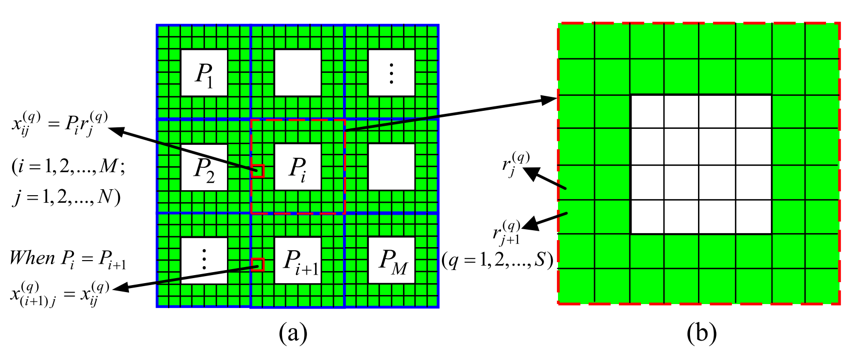

Fig. 1. Schematic figure of hierarchical optimization with multiphase materials: (a) design domain; (b) unit cell.