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An algorithm for engineering regime shifts in one-dimensional dynamical systems

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HIGHLIGHTS

- A short review of early-warning phenomena preceding bifurcations is presented.
- An algorithm for engineering regime shifts in one-dimensional dynamical systems is introduced.
- The algorithm is demonstrated on synthetic data from a couple of one-dimensional dynamical systems.

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ABSTRACT

Regime shifts are discontinuous transitions between stable attractors hosting a system. They can occur as a result of a loss of stability in an attractor as a bifurcation is approached. In this work, we consider one-dimensional dynamical systems where attractors are stable equilibrium points. Relying on critical slowing down signals related to the stability of an equilibrium point, we present an algorithm for engineering regime shifts such that a system may escape an undesirable attractor into a desirable one. We test the algorithm on synthetic data from a one-dimensional dynamical system with a multitude of stable equilibrium points and also on a model of the population dynamics of spruce budworms in a forest. The algorithm and other ideas discussed here contribute to an important part of the literature on exercising greater control over the sometimes unpredictable nature of nonlinear systems.

1. Introduction

The concept of a regime shift originated in ecology and refers to a sudden, enduring and discontinuous change in an ecosystem [1]. Regime shifts are discontinuous in the sense that they can involve large changes to the ecosystem in a short amount of time. In the context of nonlinear dynamics, regime shifts are transitions between stable attractors. Alternate stable attractors that may host the system are also known as alternate stable states in ecology. There is a large body of empirical evidence for the presence of alternate stable states and regime shifts in ecosystems [2]. The literature on regime shifts is mostly concerned with how to avoid rather than to control them [3–5]. This is because regime shifts are mostly negatively associated with unwanted phenomena like the desertification of vegetation covered regions or wildlife population collapse [6,7]. However, if one is confident of the direction of a regime shift, then engineering a regime shift can become beneficial for escaping an undesirable stable state.

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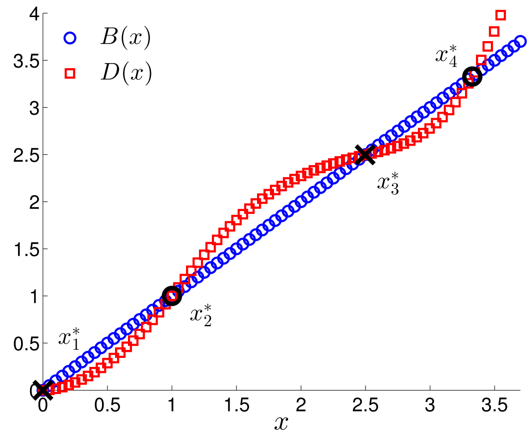

Fig. 1. Given $f(x) = B(x) - D(x)$, the equilibrium points are given by the intersections of $B(x)$ and $D(x)$ as marked by crosses (representing unstable equilibrium points) and circles (representing stable equilibrium points) on the plot.

In this paper, we will present and demonstrate an algorithm to engineer regime shifts for one-dimensional dynamical systems. A one-dimensional system can be described with

$$\dot{x} = f(x), \tag{1}$$

with x the state variable of the system. Given that $f(x)$ has N roots, each root x_i^* is an equilibrium point satisfying $f(x_i^*) = 0$ for all $0 < i \leq N$, with i and N positive integers. Let $X^* = (x_1^*, x_2^*, \ldots, x_N^*)$ represent the sequence of equilibrium point solutions to $f(x)$ such that $x_i^* > x_{i-1}^*$. Since $f(x)$ is a smooth function of x, an obvious result is that the stability of the equilibrium points in the sequence X^* always alternate between stable and unstable, not counting the half-stable equilibrium points (Proposition 1 in Appendix). In population dynamics, $f(x)$ may also be given in terms of birth and death processes, i.e. $f(x) = B(x) - D(x)$, where $B(x)$ and $D(x)$ correspond to functions representing birth and death processes respectively. Equilibrium point solutions are the intersections of $B(x)$ and $D(x)$ in this case (Fig. 1).

For the system residing at a stable equilibrium point x_s^*, the problem of engineering a regime shift to another stable equilibrium point either larger or smaller than x_s^* requires the system to overcome the basin of attraction $x_{s+1}^* - x_s^*$ or $x_s^* - x_{s-1}^*$ respectively. One way of creating a regime shift is by directly increasing or decreasing x through external means. This may not be possible or may be difficult for a large basin of attraction. Another way is to reduce the size of the basin of attraction by tuning a bifurcation parameter towards a bifurcation. Dynamic noise that is affecting the system may then cause the system to wander past the smaller basin of attraction, leading to a regime shift [8]. Alternatively, if the bifurcation parameter is tuned until a bifurcation occurs, the stable equilibrium point may change stability or get annihilated altogether, leading to a regime shift. However, there are two main problems that have to be addressed when trying to alter a bifurcation parameter to bring about a bifurcation: (1) identifying the bifurcation parameter, and (2) determining what direction the resulting regime lies in after a bifurcation has occurred. Both of these problems can be solved by measuring critical slowing down (CSD) signals in the system.

Critical slowing down signals are statistical signals arising from the phenomenon of critical slowing down, where the decay rate of perturbations to a dynamical system residing in an attractor becomes slower as the attractor approaches a bifurcation and loses stability [9,1]. These signals have been detected in a wide variety of physical, natural and socioeconomic systems on the verge of undergoing critical transitions and regime shifts [10,11,12,9]. By measuring these signals, we can tell whether a not a system is losing stability and approaching a bifurcation point. The skewness of fluctuations, itself an early warning signal to regime shifts [13], also tells us the direction of regime shifts after some types of bifurcation. Because $f(x)$ is continuous, the resulting regime (if it exists) will lie in the direction where the skewness is changing (positive for increasing skewness and negative for decreasing skewness). Fluctuations do not become skewed before a pitchfork bifurcation because the equilibrium point is symmetrically annihilated by unstable equilibrium points from both directions. However, if we approach the pitchfork bifurcation in the symmetry broken state, fluctuations do become skewed. Skewness as a direction of regime shift can also work for other types of bifurcations like the saddle–node bifurcation where there is an increasing lack of symmetry in the stability of the equilibrium point as a bifurcation point is approached. In the next section, we will go into detail on the decay rate phenomena that can be observed for the various local bifurcations before presenting and demonstrating the algorithm in Section 3. Finally, a discussion of the results in Section 4 concludes the paper.

2. Decay rate phenomena in various local bifurcations

The decay rate of a perturbation from any stable equilibrium point x^* for a continuous one-dimensional dynamical system $\dot{x} = f(x)$ is governed by $f(x)$. Let $f(x)$ be a smooth function of x. The decay rates are symmetrical between both