Physica A 486 (2017) 751–761

Contents lists available at ScienceDirect

Physica A

journal homepage: www.elsevier.com/locate/physa

Positive recurrent of stochastic coral reefs model

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HIGHLIGHTS

- We shall show a sufficient and almost necessary condition for permanence and extinction.
- We shall show a unique invariant probability measure by geometric control theory.
- It is proved that the densities converges weakly to a singular measure.

ARTICLE INFO

Article history:
Received 22 December 2016
Received in revised form 12 February 2017
Available online 15 June 2017

Keywords:
Coral reefs
Permanence
Stationary distribution
Markov semigroups
Geometric control theory

ABSTRACT

In this work, we discuss permanence and ergodicity of a stochastic coral reefs model. One of the distinctive features of our results is that our results enables characterization of the support of a unique invariant probability measure by Lie groups and geometric control theory. It is proved that the densities either converges to an invariant density or converges weakly to a singular measure.

1. Introduction

It is acknowledged that coral reefs are globally threatened. P.J. Mumby et al. [1] constructed a mathematical model with ordinary differential equations to investigate the dynamics of coral reefs. The coral reef dynamics are described as a system of coupled nonlinear ordinary differential equations [1–5]:

$$\begin{cases} \dot{X}(t) = X(t)\left(\gamma - \gamma X(t) + (\sigma - \gamma)Y(t) - \dfrac{g}{1 - Y(t)}\right), \\ \dot{Y}(t) = Y(t)\left(r - d - (\sigma + r)X(t) - rY(t)\right), \end{cases} \tag{1.1}$$

where $X(t)$ and $Y(t)$ represent, respectively, the cover of macroalgae and corals, and $\sigma < d < \gamma < r < 2\gamma,\ 0 < g < \gamma$,

- r is the rate that corals recruit to and overgrow algal turfs;
- d is the natural mortality rate of corals;
- σ is the rate that corals are overgrown by macroalgae;
- γ is the rate that macroalgae spread vegetatively over algal turfs;
- g is the grazing rate that parrotfish graze macroalgae without distinction from algal turfs.

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http://dx.doi.org/10.1016/j.physa.2017.05.073

In this paper, we consider the case of uncertainties from the recycle, namely the recycle rate takes the form as

$$\gamma \to \gamma + \alpha dW_t, \quad r \to r + \beta dW_t$$

in system (1.1). Then we are ready to propose our model as follows, which is a stochastic coral reefs model

$$\begin{cases} dX = X\left(\gamma - \gamma X + (\sigma - \gamma)Y - \dfrac{g}{1-Y}\right)dt + \alpha X dW_t, \\ dY = Y\left(r - d - (\sigma + r)X - rY\right)dt + \beta Y dW_t, \end{cases} \tag{1.2}$$

where W_t is the one dimensional Brownian motion defined on the complete probability space $(\Omega, \mathcal{F}, \{\mathcal{F}_t\}_{t\geq 0}, \mathbb{P})$ with the filtration $\{\mathcal{F}_t\}_{t\geq 0}$ satisfying the usual conditions, i.e., it is increasing and right continuous while $\mathcal{F}_0$ contains all $\mathbb{P}$-null sets. The positive numbers γ and r are the coefficients of the effect of environmental stochastic perturbation on the cover of macroalgae and corals ecosystem respectively. In this model, the random factor makes influences on the intrinsic growth rates of the cover of macroalgae and corals.

Substituting $X = e^{x_t}$, $Y = e^{y_t}$, we replace system (1.2) by

$$\begin{cases} dx = \left(\gamma - \dfrac{\alpha^2}{2} - \gamma e^{x_t} - (\gamma - \sigma)e^{y_t} - \dfrac{g}{1 - e^{y_t}}\right)dt + \alpha dW_t, \\ dy = \left(r - d - \dfrac{\beta^2}{2} - (\sigma + r)e^{x_t} - re^{y_t}\right)dt + \beta dW_t. \end{cases} \tag{1.3}$$

We interest in the ergodicity of the stochastic coral reefs model (1.2). To investigate the ergodicity for a stochastic dynamical system is important but quite challenging task in general [6–14]. Some results [15–20] in recent literature in general have been obtained by the construction of Lyapunov functionals. Although a very useful method for proving the stationary distributions, extinction and permanence. Moreover, although one may assume the existence of an appropriate Lyapunov function, it is fairly difficult to and an effective Lyapunov function in practice. Because of these difficulties, there has been no decisive classification for stochastic coral reefs models that is similar to the deterministic case. Our main goal in this paper is to provide such a classification

The organization of the paper is as follows. In Section 2, we present some preliminary results to be used in a subsequent section. In Section 3, we shall show a sufficient and almost necessary condition for permanence and extinction of the cover for stochastic coral reefs model (1.3) by using the boundary theory. In Section 4, we show that the density of the distribution of the solutions either converges to a stationary density or weakly converges to some probability measure by Lie groups, support theorems, Harris operators and geometric control theory.

2. Preliminaries and notations

In the section, we present some preliminary results to be used in a subsequent section. Let (x_t, y_t) be a solution of the system (1.3) such that the distribution of (x_0, y_0) is absolutely continuous with the density $v(x, y)$. Then the random variable (x_t, y_t) has the density $u(x, y, t)$ and u satisfies the Fokker–Planck equation:

$$\frac{\partial u}{\partial t} = \frac{1}{2}\alpha^2\frac{\partial^2 u}{\partial x^2} + \alpha\beta\frac{\partial^2 u}{\partial x \partial y} + \frac{1}{2}\beta^2\frac{\partial^2 u}{\partial y^2} - \frac{\partial(f_1(x,y))u}{\partial x} - \frac{\partial(f_2(x,y))u}{\partial y}, \tag{2.1}$$

where $f_1 = \gamma - \frac{\alpha^2}{2} - \gamma e^{x_t} + (\sigma - \gamma)e^{y_t} - \frac{g}{1-e^{y_t}}$ and $f_2 = r - d - \frac{\beta^2}{2} - (\sigma + r)e^{x_t} - re^{y_t}$.

Let $\mathcal{P}(t, x, y, A)$ is the transition probability function for the diffusion process (x_t, y_t), i.e. $\mathcal{P}(t, x, y, A) = \mathrm{Prob}((x_t, y_t) \in A)$ and (x_t, y_t) is a solution of the system (1.3) with the initial condition (x_0, y_0).

For each point $(x, y) \in \Re^2$ and $t > 0$, we check that the measure $\mathcal{P}(t, x, y, A)$ is absolutely continuous with respect to the Lebesgue measure. Thus the distribution of any solution (x_t, y_t) of the system (1.3) is absolutely continuous for $t > 0$ and its density u satisfies (2.1).

Now we introduce some results concerning Markov semigroups we will use later.

Let the triple (Z, Λ, m) be a σ-finite measure space. D denote the subset of the space $L^1 = L^1(Z, \Lambda, m)$ which contains all densities, i.e.

$$D = \left\{f \in L^1 : f \geq 0, \|f\| = 1\right\}.$$

A linear mapping $P : L^1 \to L^1$ is called a Markov operator if $P(D) \subset D$.

The Markov operator P is called an kernel operator or integral if there exists a measurable function $k : Z \times Z \longrightarrow [0, \infty)$ such that

$$Pf(x) = \int_Z k(x, y)f(y)m(dy)$$

for every density f. From the condition $P(D) \subset D$, one can check that

$$\int_Z k(x, y)m(dy) = 1 \tag{2.2}$$

for almost all $y \in Z$.