



S-Lagrangian dynamics of many-body systems and behavior of social groups: Dominance and hierarchy formation



U. Sandler

Lev Academic Center (JCT), Jerusalem 91160, Israel

HIGHLIGHTS

- Many-body system with S-dependent Lagrangian inevitably develops a hierarchical structure.
- Lagrangian and energy of these systems are vector functions, while conjugated momenta are second-order tensors.
- Dominance hierarchies are determined by differences between individuals in coping with stress an individual with fastest coping with stress becomes the dominant.

ARTICLE INFO

Article history:

Received 1 March 2017

Received in revised form 18 May 2017

Available online 3 June 2017

Keywords:

S-Lagrangian

Dynamics of complex systems

Living systems

Social groups

Dominance

Hierarchy formation

ABSTRACT

In this paper, we extend our generalized Lagrangian dynamics (i.e., S-Lagrangian dynamics, which can be applied equally to physical and non-physical systems as per Sandler (2014)) to many-body systems. Unlike common Lagrangian dynamics, this is not a trivial task. For many-body systems with S-dependent Lagrangians, the Lagrangian and the corresponding Hamiltonian or energy become vector functions, conjugated momenta become second-order tensors, and the system inevitably develops a hierarchical structure, even if all bodies initially have similar status and Lagrangians. As an application of our theory, we consider dominance and hierarchy formation, which is present in almost all communities of living species. As a biological basis for this application, we assume that the primary motivation of a groups activity is to attempt to cope with stress arising as pressure from the environment and from intrinsic unmet needs of individuals. It has been shown that the S-Lagrangian approach to a group's evolution naturally leads to formation of linear or despotic dominance hierarchies, depending on differences between individuals in coping with stress. That is, individuals that cope more readily with stress take leadership roles during the evolution. Experimental results in animal groups which support our assumption and findings are considered.

© 2017 The Author. Published by Elsevier B.V. This is an open access article under the CC BY license (<http://creativecommons.org/licenses/by/4.0/>).

0. Introduction

In the study of real complex systems (for example, living organisms) most system parameters remain hidden or out of control. This leads to large deviations in experimental results. Consequently, small differences in the numerical values of experimental data lose their significance. Indeed, the state of such a system is better described by a *domain* of points rather than a *single* point in the state space of the system. This kind of uncertainty does not have a stochastic nature, and following the idea proposed by Zadeh [1–3], we could describe the *possibility* of a given state of a system by some function $0 \leq \mu(r_1 \dots r_N; t) \leq 1$ of the systems parameters $\{r_1 \dots r_N\}$ and time t and the *possibility* of a system's "moving" by some

E-mail address: sandler@gjct.ac.il.

<http://dx.doi.org/10.1016/j.physa.2017.05.055>

0378-4371/© 2017 The Author. Published by Elsevier B.V. This is an open access article under the CC BY license (<http://creativecommons.org/licenses/by/4.0/>).

function of the rates of change of the system’s parameters, $0 \leq \mathcal{P}(\dot{r}^1 \dots \dot{r}^N; r_1 \dots r_N) \leq 1$ (later we will call \dot{r}^k “velocities for brevity”).

It has been shown in [4,5] that, if a system’s evolution satisfies the causality principle, if the system’s state space has a trivial local topology, and if the system’s state can be described by a compact fuzzy set,¹ then the function $\mu(r_1 \dots r_N; t)$ satisfies the equation²

$$\frac{\partial \mu}{\partial t} + \left(\dot{r}^k \cdot \frac{\partial \mu}{\partial r_k} \right) = 0, \tag{1}$$

where \dot{r}^k should be found from:

$$\lambda \frac{\partial \mathcal{P}}{\partial \dot{r}^k} = \frac{\partial \mu}{\partial r_k}, \tag{2}$$

$$\mathcal{P}(\dot{r}^1 \dots \dot{r}^N; r_1 \dots r_N) = \chi(\mu), \tag{3}$$

where $\chi(\mu)$ is a monotonic function with $\chi(1) = 1$ and $\chi(0) = 0$, and $\lambda > 0$ is a Lagrange multiplier. It should be emphasized that the derivation of Eqs. (1)–(3) does not depend on any specific properties of a system (see [4,5] for details).

In [4], single-body systems were considered, and it has been shown that the most possible system trajectories³ satisfy the generalized Lagrangian-like equations

$$\frac{d}{dt} \frac{\partial L}{\partial \dot{q}_i} - \frac{\partial L}{\partial q_i} = \frac{\partial L}{\partial S} \frac{\partial L}{\partial \dot{q}_i}, \tag{4a}$$

$$\frac{dS}{dt} = L(\dot{\mathbf{q}}, \mathbf{q}, S, t), \tag{4b}$$

where $L(\dot{\mathbf{q}}, \mathbf{q}, S, t)$ is a solution of (3) with respect to the velocity \dot{S} of some parameter S , while $\{\mathbf{q}, \dot{\mathbf{q}}\}$ are the remaining parameters and their velocities. The Lagrangian $L(\dot{\mathbf{q}}, \mathbf{q}, S, t)$ was called the “most possible S-Lagrangian (or the S-Lagrangian, for brevity).

In Section 1 we generalize this approach to many-body systems. Unlike in the common classical mechanics, this is not a trivial task. For many-body systems with an S-dependent Lagrangian, the Lagrangian (along with the corresponding Hamiltonian or energy) becomes a vector function, conjugated momenta become second-order tensors, and the system inevitably develops a hierarchical structure, even if all bodies initially have similar status and Lagrangians.

In Section 2, we apply the dynamics of many-body systems with S-dependent Lagrangians to explain dominance hierarchies in social groups. This phenomenon is present in almost all communities of living species, from populations of cancer cells [6] and neurons in the brain (Ukhtomsky dominanta [7]) to high levels of human societies [8,9].

There are several forms of dominance hierarchies, but the most common are linear dominance and despotic dominance. In a linear hierarchy, each individual dominates the individuals who are below him and not those above him. In a despotic hierarchy, one individual is dominant in the group and all the other members are that individuals submissive servants, while “lateral” interactions between members are irrelevant. There is sea of literature related to dominance hierarchies (see [9] and references therein). In most models, the origin of hierarchical structures is explained by competition for better resources and breeding, which leads to aggressive interactions between agents in the society. Agents that win most of their fights rise higher within the hierarchy, while those that lose most fights occupy the lowest positions (see, for example, [10] and references therein). Mathematical models of dominance hierarchies describe “winner–loser” strategy by using stochastic mechanisms and analyze this behavior numerically or, sometimes, analytically [11,12]. It should be noted, however, that in situations with an abundance resources and spare breeding ability,⁴ there are still dominance hierarchies, so other mechanisms for hierarchical structure formation are required.

Behavior of groups with non-fighting hierarchies has been also considered. The most popular example is the Cucker–Smale model of flocking with hierarchical leadership (see for example, [13,14]). In these studies, however, the hierarchy structure was static and was primordially introduced.

In spite of the differences between living species and the differences in their behavior, dominance hierarchies are strikingly similar in almost all communities. Such tremendous diversity of system features accompanied with similar properties of dominance hierarchies leads us to think that dominance hierarchies result as very general features of living beings. Hierarchy formation is based on the notion of “adaptation energy”, which was introduced by Selye [15], was considered in [16,17]. These works are conceptually close to ours, but use another mathematical and biological approaches.

A primary difference between living creatures and non-living things is the capacity for reproduction. However, if one considers only individual life rather than the existence of species, a major paradox is that living things actively counteract degradation or injury in a continuously changing environment through homeostatic protection. The discoverer of homeostasis, Cannon [18], assumed that homeostasis results from a tendency of organisms to decrease their level of stress,

¹ That is valid for almost all real systems.

² Summation with respect to the same ascending and descending indices is assumed unless an opposite one is explicitly written.

³ These correspond to the case $\mu(r_1 \dots r_N; t) = 1$, $\mathcal{P}(\dot{r}^1 \dots \dot{r}^N; r_1 \dots r_N) = 1$.

⁴ An example could be a case in which there is a small number of males and a large number of females in a friendly environment.

Download English Version:

<https://daneshyari.com/en/article/5102596>

Download Persian Version:

<https://daneshyari.com/article/5102596>

[Daneshyari.com](https://daneshyari.com)