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Stochastic periodic solution for a perturbed non-autonomous predator–prey model with generalized nonlinear harvesting and impulses

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HIGHLIGHTS

- Stochastic predator–prey models with impulsive effects are investigated.
- Generalized nonlinear harvesting is also considered in the model.
- Sufficient conditions for extinction and persistence in the mean are achieved.
- Global attractiveness and stochastic persistence in probability are discussed.
- The existence of periodic solutions of the two systems is obtained.

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ABSTRACT

In this paper, stochastic non-autonomous predator–prey models with and without impulses are investigated. The effects of generalized nonlinear harvesting for prey and predator populations are considered. For the stochastic system without impulses, the existence and uniqueness of the positive solution is proven and sufficient conditions that guarantee the extinction and persistence of the population in the mean are achieved. We show the existence of a nontrivial positive periodic solution by constructing appropriate Lyapunov functions and using Khasminskii's theory. Moreover, the global attractiveness and stochastic persistence in probability of the stochastic model are discussed. Results show that the stronger noises and nonlinear harvesting component can significantly influence the dynamics of the system and lead to the extinction of the predator population. Additionally, for the stochastic predator–prey system with impulsive effect, we prove that there exists a positive periodic solution. Numerical simulations are conducted to show the effectiveness and feasibility of the obtained results.

1. Introduction

Population dynamics has attracted considerable interest from many scientific communities, including those of biology, ecology, and economics [1]. Given the importance of renewable resource management, a harvesting component is increasingly being incorporated into mathematical models to reveal its effects on one or multiple species [2–5]. The commonly used harvesting functions are constant harvesting, proportional harvesting, and nonlinear harvesting.

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Xiao et al. [6] considered the effects of constant harvesting on predator species and discussed the dynamical properties of the following model:

$$\begin{cases} \dfrac{dx}{dt} = x(1-x) + \dfrac{axy}{y+x}, \\ \dfrac{dy}{dt} = y\left(-d + \dfrac{bx}{y+x}\right) - h, \end{cases} \tag{1.1}$$

where $x(t)$ and $y(t)$ are the prey and predator population densities, respectively. They also showed that the system presents two equilibria at most in the first quadrant. Saddle–node and Hopf bifurcations were detected as well. Lenzini and Rebaza [1] extended the constant predator harvesting to nonconstant functions on a ratio-dependent model. Moreover, they discussed the stability properties, boundedness of solutions, and bifurcations. Gupta et al. [7] introduced a predator–prey model with nonlinear predator harvesting, which was closer to reality than other harvesting types:

$$\begin{cases} \dfrac{dx}{dt} = x(a_1 - b_1 x) + axy, \\ \dfrac{dy}{dt} = -dy + \eta axy - \dfrac{hy}{1+by}. \end{cases} \tag{1.2}$$

The existence and global stability of the equilibria were investigated and the effects of the harvesting function on the rich dynamics of their model were analyzed. Considering the influence of environmental noise, Zuo and Jiang [8] proposed a stochastic predator–prey model with nonlinear harvesting

$$\begin{cases} dx(t) = \Big(x(a_1 - b_1 x) - axy\Big)dt + \alpha x dB_1(t), \\ dy(t) = \Big(-dy + \eta axy - \dfrac{hy}{1+by}\Big)dt - \beta y dB_2(t), \end{cases} \tag{1.3}$$

and discussed the stationary distribution of the model. In recent studies, several authors (see e.g. [9–12]) concentrated on the effects of stochastic perturbations and harvesting. However, in their models, the harvesting function was considered only for the predator, and not for the prey.

On the other hand, impulsive differential equations are highly effective methods for describing species and ecological systems, realistically. A number of peculiar results have been obtained regarding dynamic behavior of stochastic systems, including the permanence, extinction and dynamic complexity, see [13,14]. However, only a few studies have addressed and investigated the existence of positive periodic solutions of the stochastic predator–prey model, with impulsive perturbation and generalized nonlinear harvesting.

Stimulated by the above review of literature, we consider the effects of impulses and the generalized nonlinear harvesting rates on the prey and predator populations, and propose the following two interesting stochastic non-autonomous systems:

$$\begin{cases} dx(t) = x\Big(r(t) - k(t)x - b(t)y - \dfrac{e(t)}{f(x(t))}\Big)dt + \sigma_1(t) x dB_1(t), \\ dy(t) = y\Big(-d(t) - a(t)y + \eta(t)b(t)x - \dfrac{h(t)}{g(y(t))}\Big)dt - \sigma_2(t) y dB_2(t) - \dfrac{\sigma_3(t)h(t)y}{g(y(t))} dB_3(t), \end{cases} \tag{1.4}$$

and

$$\begin{cases} \left.\begin{aligned} dx(t) &= x\Big(r(t) - k(t)x - b(t)y - \dfrac{e(t)}{f(x(t))}\Big)dt + \sigma_1(t) x dB_1(t) \\ dy(t) &= y\Big(-d(t) - a(t)y + \eta(t)b(t)x - \dfrac{h(t)}{g(y(t))}\Big)dt - \sigma_2(t) y dB_2(t) - \dfrac{\sigma_3(t)h(t)y}{g(y(t))} dB_3(t) \end{aligned}\right\} \quad t \neq t_i, \\ \left.\begin{aligned} x(t_i^+) &= (1+\alpha_i)x(t_i) \\ y(t_i^+) &= (1+\beta_i)y(t_i) \end{aligned}\right\} \quad t = t_i, \quad i = 1, 2, 3, \dots \end{cases} \tag{1.5}$$

where $r(t)$ and $d(t)$ are the respective intrinsic growth rates for the prey and predator species, $b(t)$ is the capture rate of the predator, $\eta(t)$ represents the conversion rate of the nutrients into a new predator, and $k(t)$ and $a(t)$ are the density-dependent coefficients of the prey and predator species, respectively. In addition, $f(x)$ and $g(y)$ are continuously differentiable functions that are strictly monotone on $[0, +\infty)$ with $f(0) = g(0) = 1$ and $f(x) \le x$, $g(y) \le y$ for all $x > 0$ and $y > 0$. We suppose that $(\Omega, \mathcal{F}, \{\mathcal{F}_t\}_{t\ge 0}, \mathbb{P})$ is a complete probability space with a filtration $\{\mathcal{F}_t\}_{t\ge 0}$ satisfying the usual conditions, that is, it is right continuous and increasing, while $\mathcal{F}_0$ contains all $\mathbb{P}$-null sets.

In this study, all the coefficients are assumed to be positive θ-periodic continuous functions and bounded on $\mathbb{R}_+ = [0, +\infty)$. The impulsive points satisfy $0 < t_1 < t_2 < \cdots < t_i < \cdots$ and $\lim_{i\to+\infty} t_i = +\infty$. According to biological meanings, $\alpha_i > -1$, $\beta_i > -1$. Moreover, we assume that a integer $p > 0$ exists satisfying $t_{i+p} = t_i + \theta$, $\alpha_{i+p} = \alpha_i + \theta$, $\beta_{i+p} = \beta_i + \theta$, $i \in \mathbb{Z}$ and $[0, \theta) \cap \{t_i, i \in \mathbb{Z}\} = \{t_1, t_2, \dots, t_p\}$. The aim of this paper is to prove the existence of the stochastic