



Three-state Potts model on non-local directed small-world lattices

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HIGHLIGHTS

- $q = 3$ Potts model on NLDSW lattices has a first-order phase transition for $p > p^* = 0.05$.
- $q = 3$ Potts model on NLDSW lattices is a different universality class for $0.01 \leq p \leq p^*$.
- We estimate the critical temperatures of the $q = 3$ Potts model for $0.01 \leq p \leq p^*$.

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ABSTRACT

In this paper, we study the non-local directed Small-World (NLDSW) disorder effects in the three-state Potts model as a form to capture the essential features shared by real complex systems where non-locality effects play a important role in the behavior of these systems. Using Monte Carlo techniques and finite-size scaling analysis, we estimate the infinite lattice critical temperatures and the leading critical exponents in this model. In particular, we investigate the first- to second-order phase transition crossover when NLDSW links are inserted. A cluster-flip algorithm was used to reduce the critical slowing down effect in our simulations. We find that for a NLDSW disorder densities $p < p^* = 0.05(4)$, the model exhibits a continuous phase transition falling into a new universality class, which continuously depends on the value of p , while for $p^* \leq p \leq 1.0$, the model presents a weak first-order phase transition.

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1. Introduction

In the past, the connection topology had been assumed to be either completely random or completely regular. But many biological, technological and social networks lie somewhere between these two extremes. The small-world (SW) topology [1] is suitable for this purpose and constitutes an interesting attempt to translate complex networks as physical, biological and social networks into a simple model. Applications to earth sciences, brain sciences, computing and sociology have been extensively reported. Remarkably, dynamical systems with small-world coupling exhibit enhanced signal-propagation speed [2] and synchronizability [3] when compared to systems with regular coupling. SW networks are obtained by randomly replacing a fraction p of links of a regular lattice with new random links. As a result of this random rewiring, SW networks interpolate between a regular lattice $p = 0$ and a completely random graphs $p = 1$.

Two different types of SW networks have been purposed to understand the underlying features found in real complex networks: Undirected (standard) SW networks [1] and directed SW networks [4]. Undirected SW networks are formed by symmetric links in the sense that if a given node X of the network is linked to a given node Y , then the node Y must also be

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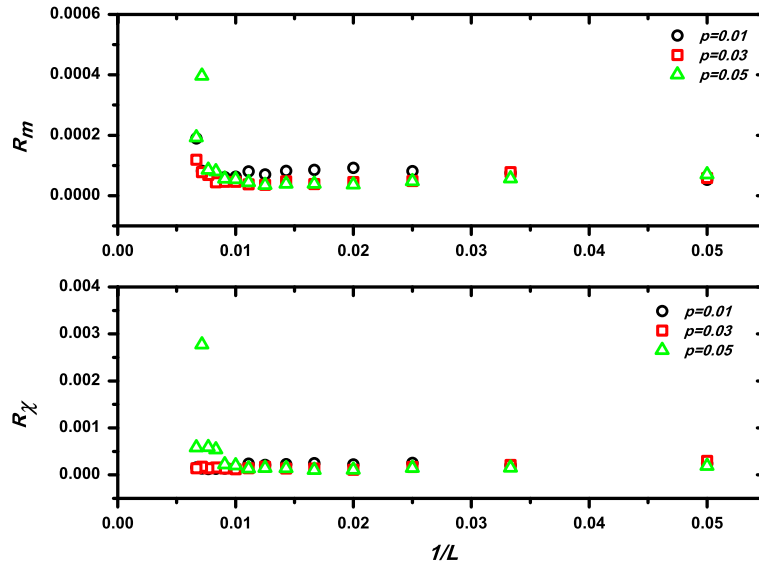


Fig. 1. Plot of the relative variance for both the magnetization R_m (top panel) and the susceptibility R_χ (bottom panel) versus the inverse of the lattice size at the effective critical temperature $T_c(L)$ (as given by Eq. (11)) for three studied values of the disorder density p .

linked to the node X , such as in the network of movie actors' collaboration or authorship of scientific papers. While directed SW networks are formed by asymmetric links, i.e., if a given node X of the network is linked to a given node Y , the node Y may not be linked to X but to another. Such examples of this kind of network are World Wide Web, lending transactions and asymmetric synaptic connections.

Although spin-like models on SW topology has been intensely studied in last two decades, there have rarely been studies concerning directed SW topology where only non-local connections take place in these models. Knowing how local and non-local SW directed disorder separately influence the dynamics of systems is particularly important. Moreover, the criticality in systems with this kind of connection disorder remains little known. Therefore, this issue necessitates extensive computational research for accurately estimating the static critical exponents in these systems.

While earlier studies [5–7] employing directed SW network in different spin models have showed that this connection disorder can change the universality class of these models. However, they have not distinguishably focused on non-local directed SW disorder. But many systems found in the nature exhibit non-locality effects in which long-range interactions entirely dominate the time evolution of these systems. Such examples are sexually transmitted diseases, genetic recombination and quantum computation, among others.

In this paper we study the effects of the non-local directed Small-World (NLDSW) disorder in the three-state Potts model as form to capture the essential features shared by real complex systems where non-locality effects play a important role in the behavior of these systems. We choose the ferromagnetic Potts model [8,9] because its simplicity and well-known phase transition properties. Using Monte Carlo (MC) techniques [10–12] and finite-size scaling (FSS) analysis, we estimate the infinite lattice critical temperatures and the leading critical exponents in this model. In particular, we investigate the first-to second-order phase transition crossover when non-local directed links are inserted. In order to reduce the critical slowing down effect [13], we used a cluster-flip algorithm to evolve the lattices studied over time. Periodic boundary conditions were also used to avoid the boundary effects caused by the finite size. In this study, we make an analysis of several thermodynamic quantities including the specific heat and susceptibility as well as the fourth-order energetic cumulant, derivatives and logarithmic derivatives of the magnetization.

The contents of the article are organized as follow. In Section 2, we describe details of the model and Monte Carlo simulation background. In Section 3, we present and discuss the results. Finally, in Section 4, we make the conclusions.

2. Model and Monte Carlo simulation

The NLDSW lattices used in MC simulations were constructed in similar way as in Sánchez *et al* [4]. First, we start from a regular square lattice consisting of sites linked to their four nearest neighbors by both outgoing and incoming links. Then, with probability p , we reconnect every nearest-neighbor outgoing link to a new site randomly chosen provided that it is neither the site itself (self-interaction) nor any of its four nearest neighbor (local interaction). After repeating this procedure for every outgoing link, a new lattice is constructed with a density p of NLDSW links. In this lattice, each site will have four outgoing links but a random number of incoming links. To study the critical behavior in NLDSW lattices, we use a Wolff algorithm [14] to update the lattices studied. For a fixed temperature, we define a Monte Carlo step (MCS) per spin by

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