Physica A 484 (2017) 336–344

Contents lists available at ScienceDirect

Physica A

journal homepage: www.elsevier.com/locate/physa

Fixation of strategies with the Moran and Fermi processes in evolutionary games

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HIGHLIGHTS

- The one-third rule holds for our model.
- Whether fixation occurs more easily than do Moran process depends on the game.
- Compared with the standard Moran process, fixation always takes more time.
- A defective mutant can take over slower when the number of neighbors is smaller.

ARTICLE INFO

Article history:
Received 29 July 2016
Received in revised form 24 April 2017
Available online 8 May 2017

Keywords:
Evolution of cooperation
Fixation probability
Fixation time
One-third rule

ABSTRACT

A model of stochastic evolutionary game dynamics with finite population was built. It combines the standard Moran and Fermi rules with two strategies cooperation and defection. We obtain the expressions of fixation probabilities and fixation times. The one-third rule which has been found in the frequency dependent Moran process also holds for our model. We obtain the conditions of strategy being an evolutionarily stable strategy in our model, and then make a comparison with the standard Moran process. Besides, the analytical results show that compared with the standard Moran process, fixation occurs with higher probabilities under a prisoner's dilemma game and coordination game, but with lower probabilities under a coexistence game. The simulation result shows that the fixation time in our mixed process is lower than that in the standard Fermi process. In comparison with the standard Moran process, fixation always takes more time on average in spatial populations, regardless of the game. In addition, the fixation time decreases with the growth of the number of neighbors.

1. Introduction

Cooperation is a widespread phenomenon studied extensively in social, economical and biological sciences. Cooperative behavior can be found in both animal and human societies. The evolution of cooperation is a major issue in the study of evolutionary biology since Darwin [1–7]. Evolutionary game theory has provided a powerful framework to discuss this issue in a quantitative manner [8–17]. Applications of this framework can also describe the dynamics of human behavior [18–23]. In order to understand how cooperation can be favored, five rules for the evolution of cooperation have been obtained by Nowak [24], including kin selection, direct reciprocity, indirect reciprocity, network reciprocity, and group selection. Some further theoretical analyses in both infinite and finite populations have been achieved. In infinite populations, evolutionary

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http://dx.doi.org/10.1016/j.physa.2017.04.154
0378-4371/

game dynamics is studied by deterministic differential equations, while, in finite populations, evolutionary game dynamics provides a new framework to study selection of traits with frequency-dependent fitness [25–30]. The prisoner's dilemma (PD) game which has attracted great interest in the last decades is an excellent model to describe the ways in which the cooperative behavior is maintained among selfish individuals. The original PD game is a two-person one-shot game where individuals simultaneously choose one of two strategies: cooperation and defection. Assuming a symmetric game two individuals receive the "reward" R if both are cooperators, while two defectors are punished to get the "punishment" P which is the lowest payoff. A cooperator receives the "sucker" payoff S against a defector, while a defector gets the "temptation" T against a cooperator. The rank of the four payoff values is arranged as $T > R > P > S$, so that the defector will gain a fitness advantage compared to a cooperative one.

Microscopic processes are often used to determine how strategies spread in finite populations. Two kinds of microscopic processes are used extensively: Moran process and Fermi process. In the standard Moran process, an individual chosen proportional to fitness produces one individual offspring and the offspring replaces a randomly chosen individual [31–33]. In the standard Fermi process, a randomly chosen individual evaluates its success by comparing its payoff with a second, randomly chosen individual [34,35]. [36] analyzed how universal weak selection for those two microscopic processes is. [37] demonstrated the findings of fixation times for those two microscopic processes. [38] provided further consideration on those microscopic processes. [39] is paper introduces a combination of the standard Moran and Fermi processes. The mixed process can make sense under the following condition.

We assume that there exist two kinds of human behavior when human beings are at work: some of them who work hard are called cooperators, and they will share the achievements of labor to other working partners, while others who do not work in earnest are called defectors, they wait to get something for nothing. Humans are regarded as cooperators or defectors when they begin to work for a task. And subsequently there exists one chance to change its strategy by imitating other human (in our model, the probability to imitate successfully is given by the Fermi distribution).

This paper is organized as follows. We introduce a particular evolutionary process mixed with the standard Moran process and Fermi process in Section 2. Sections 3 and 4 offers some analysis of fixation probabilities and fixation times, respectively. We obtain a conclusion of our results in Section 5.

2. Model

In a well-mixed population of size N, individuals play a symmetric 2×2 game between two strategies C (cooperation) and D (defection), which can be described by the payoff matrix:

$$\begin{array}{cc} & \begin{array}{cc} C & D \end{array} \\ \begin{array}{c} C \\ D \end{array} & \left(\begin{array}{cc} a & b \\ c & d \end{array} \right) \end{array}. \tag{1}$$

A cooperator interacting with another cooperator receives a, while a defector interacting with another defector receives d. When a cooperator interacts with a defector, the cooperator obtains a, whereas the defector obtains c.

At each Monte Carlo step, an individual X is chosen proportional to fitness to produce one offspring X' with the same strategy. To keep the size of the population constant, a randomly chosen individual Y is removed from the population before the offspring added. Then the offspring X' evaluates its success by comparing its payoff with a second, randomly chosen individual Z. X' imitates Z with probability $p_{X' \to Z} = 1/(1 + e^{\omega(\pi_{X'} - \pi_Z)})$, where ω is the intensity of selection and the payoffs of X' and Z are described by $\pi_{X'}$ and π_Z, respectively. For a sketch of the evolutionary game dynamics model, see Fig. 1.

The average payoffs of a cooperator and a defector are then given by:

$$\pi_C = \frac{a\,(i-1) + b\,(N-i)}{N-1}, \tag{2a}$$

$$\pi_D = \frac{ci + d\,(N-i-1)}{N-1}. \tag{2b}$$

The number of cooperators i denotes the state of the system. The two pure states $i = 0$ and $i = N$ are absorbing. We denote fitness for cooperators and defectors by f_C and f_D, respectively. Additionally, we use an exponential mapping [18]:

$$f_C = e^{\omega \pi_C}, \tag{3a}$$

$$f_D = e^{\omega \pi_D}. \tag{3b}$$

In the mixed process, the transition probabilities are given by:

$$\begin{aligned} T_i^+ \;=\; & \frac{ie^{\omega\pi_C(i)}}{ie^{\omega\pi_C(i)} + (N-i)\,e^{\omega\pi_D(i)}} \frac{N-i}{N} \left(1 - \frac{N-i-1}{N} \frac{1}{1 + e^{\omega(\pi_C(i+1) - \pi_D(i+1))}} \right) \\ & + \frac{(N-i)\,e^{\omega\pi_D(i)}}{ie^{\omega\pi_C(i)} + (N-i)\,e^{\omega\pi_D(i)}} \frac{N-i}{N} \frac{i}{N} \frac{1}{1 + e^{\omega(\pi_D(i) - \pi_C(i))}}, \end{aligned} \tag{4a}$$