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Eigentime identities for random walks on a family of treelike networks and polymer networks

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HIGHLIGHTS

- The eigentime identity of networks is studied.
- The Sierpinski networks and polymer networks are introduced.
- Computing the roots of several small-degree polynomials.
- Applying the spectral decimation approach to determine the normalized Laplacian spectra.

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ABSTRACT

In this paper, we investigate the eigentime identities quantifying as the sum of reciprocals of all nonzero normalized Laplacian eigenvalues on a family of treelike networks and the polymer networks. Firstly, for a family of treelike networks, it is shown that all their eigenvalues can be obtained by computing the roots of several small-degree polynomials defined recursively. We obtain the scalings of the eigentime identity on a family of treelike with network size N_n is $N_n \ln N_n$. Then, for the polymer networks, we apply the spectral decimation approach to determine analytically all the eigenvalues and their corresponding multiplicities. Using the relationship between the generation and the next generation of eigenvalues we obtain the scalings of the eigentime identity on the polymer networks with network size N_n is $N_n \ln N_n$. By comparing the eigentime identities on these two kinds of networks, their scalings with network size N_n are all $N_n \ln N_n$.

1. Introduction

In the past decade, the study of networks associated with complex systems has received the attentions of researchers from different scientific fields [1–7]. The eigentime identity of multi-agent systems has gained much interest [8,9].

Julaiti et al. mentioned that the sum of reciprocals of each nonzero eigenvalues of normalized Laplacian matrix for a network determines the eigentime identity of random walks on the network, which is a global characteristic of the network, and reflects the architecture of the whole network [9].

In this paper, we first use normalized Laplacian spectrum to get the eigentime identity on a family of treelike networks. For a family of treelike networks, we show that all their eigenvalues can be obtained by computing the roots of several small-degree polynomials defined recursively.

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Then for the polymer networks, we apply the spectral decimation approach to determine analytically all the eigenvalues and their corresponding multiplicities, with the eigenvalues provided by a recursive relation governing the eigenvalues of networks at two successive generations.

The organization of this paper is as follows. In next section we introduce the definition of eigentime identity. In Section 3, we give the model of a family of treelike networks and get the scalings of eigentime identity with network size on a family of treelike networks. In Section 4, we give the model on the polymer networks and get the scalings of eigentime identity with network size on the polymer networks. In Section 5 we draw the conclusions.

2. Eigentime identity

In this section, we introduce the concept of eigentime identity.

Let $F_{ij}(n)$ be mean-first passage time from node i to node j in G_n, which is the expect time for a particle starting off from node i to arrive at node j for the first time. Let $d_i(n)$ be the degree of node i and E_n be the number of edges in G_n. The stationary distribution for random walks on G_n is $\pi = (\pi_1, \pi_2, \dots, \pi_N)^{\mathrm{T}}$, where $\pi_i = \frac{d_i(n)}{2E_n}$, obeying relations $\sum_{i=1}^{N} \pi_i = 1$ and $\pi^{\mathrm{T}} M_n = \pi^{\mathrm{T}}$, where M_n be the Markov matrix of G_n. Let H_n represent the eigentime identity for random walks on G_n, which is defined as the expected time for a walker going from a node i to another node j, chosen randomly from all nodes accordingly to the stationary distribution [8]. That is,

$$H_n = \sum_{j=1}^{N_n} \pi_j F_{ij}(n),$$

where N_n is the number of nodes of G_n. H_n quantifies the expected time taken by a particle starting from node i to get to a node (target) j randomly chosen according the stationary distribution. Since H_n do not rely on the starting node, it can be rewritten as

$$H_n = \sum_{i=1}^{N_n} \pi_i \sum_{j=1}^{N_n} \pi_j F_{ij}(n) = \sum_{j=1}^{N_n} \pi_j \sum_{i=1}^{N_n} \pi_i F_{ij}(n).$$

The rightmost expression in above equation indicates that the eigentime identity H_n is actually the average trapping time of a special trapping problem, which involves a double weighted average: the former ia over all the source nodes to a given trapping (target) node j. The later is the average with respect to the first one taken over the stationary distribution. Because trapping is a fundamental mechanism for various other dynamical processes, H_n contains much information about trapping and diverse processes taking place on complex systems [8].

According to previous results [8], let L_n be the normalized Laplacian matrix of G_n. H_n can be expressed in terms of the nonzero eigenvalues of L_n as

$$H_n = \sum_{i=1}^{N} \frac{1}{\lambda_i}, \tag{1}$$

where we assume $\lambda_1 = 0$.

In the following, we introduce a family of treelike networks and the polymer networks inspired by the models in [10–14]. According to their constructions, we will obtain the scalings of eigentime identity for a family of treelike networks with network size N_n and the scalings of network coherence for polymer networks with network size N_n.

3. Eigentime identity of a family of treelike networks

3.1. A family of treelike networks

In this subsection a family of treelike networks are introduced.

Let s ($s > 1$) be a positive integer. Initially ($n = 0$), $G_0^{(s)}$ consists of only a central node. To form $G_1^{(s)}$, we create s nodes and attach them to the central node. For any $n \geq 1$, $G_n^{(s)}$ is obtained from $G_{n-1}^{(s)}$ by performing the following operations. For each outermost node of $G_{n-1}^{(s)}$, s nodes are generated and attached them to the outermost node. Let $G_n^{(s)}$ be its associated network. In Fig. 1, we schematically illustrate the process of the first three iterations. From the construction of a family of treelike networks, one can see that $G_n^{(s)}$, is characterized by two parameters n and s. Let $N_i(n)$ denote the number of nodes in $G_n^{(s)}$, which are given birth to at iteration i. It is easy to check that $N_i = s^i$. The total number of nodes in $G_n^{(s)}$, N_n, satisfies the following relationship,

$$N_n = \sum_{i=0}^{n} N_i = \frac{s^{n+1} - 1}{s - 1}.$$