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# Strategies involving the local defect correction multi-level refinement method for solving three-dimensional linear elastic problems

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ABSTRACT

The aim of this study was to assess the efficiency of the local defect correction multi-grid method (Hackbusch, 1984 [31]) on solid mechanics test cases showing local singularities and derived from an industrial context. The levels of local refinement are automatically obtained recursively, using Zienkiewicz and Zhu's *a posteriori* error estimator. Choices of the prolongation operator, the refinement ratio and criterion are discussed in order to give the most satisfactory performances. Comparisons with an h-adaptive refinement method show the efficiency of the tool presented here, in terms of its accuracy and the memory space and processor time required.



## 1. Introduction

Industrial simulations involve increasingly complex physics, generally related to various characteristic length-scales. The following two main approaches have been developed to generate locally refined meshes with fewer degrees of freedom (DoF) instead of adopting a uniformly refined mesh matching the finest singularity:

- The methods of the first kind, which are known as "adaptive methods", consist in locally enriching recursively an initial mesh of the domain. As a consequence, the initial mesh has not to be fitted to the singularities. There exist four main adaptive refinement methods: the r-adaptive technique (e.g. [1,2]), the h-adaptive technique (e.g. [3–7]), the p-adaptive technique (e.g. [8–10]) and the s-adaptive technique (e.g. [11,12]). The aim of the first three approaches is to reduce the discretisation error by making local changes in either the position of the nodes (in the case of the r-adaptive method) or the number of DoF (in that of the h-adaptive method) or the degree of the polynomial basis functions (in that of the p-adaptive method). The main advantage of these latter adaptive methods is that the problem is eventually solved on a single optimum mesh. These methods are the most performed these days, especially in combined versions giving the advantages of two or more methods, such as the hr-adaptive process [13,14] or the hp-adaptive process presented in [15–17], which is widely used in the fields of thermo-hydraulics [18,19], combustion [20], neutronics [21,22] and solid mechanics [23,24], for example.
However, some extra work on the solver is usually required (non-conforming meshes, preconditioning, etc.), and the resulting number of DoF in the problem may still be prohibitive in industrial contexts.
The s-adaptive method, which is slightly different from the other approaches, consists in overlaying the initial mesh with additional finer local meshes. A composite problem is then defined, accounting for the behaviour of each mesh and the interface coupling between the connected levels. The resulting number of DoF quickly becomes huge, which explains why this method is less widespread than the others of its kind. However, a version called the Arlequin method (e.g. [25,26]), in which domains with different behaviours (for example, 1-D and 2-D models or continuum and atomic models) are combined, has recently become quite popular in the field of solid mechanics.
- The methods of the second kind, which are called local multi-grid (or multi-level) methods (e.g. [27–29]), can be seen as s-adaptive methods where the problems defined on each grid are solved separately. A multi-grid process [30] based on prolongation and restriction operators enables the solutions obtained at each level to be connected to each other. Unlike the standard multi-grid approach, the local multi-grid procedure starts with a coarse overall grid (which is generally

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non-data-fitted) covering the whole domain and then works recursively on fine local nested sub-grids. Each mesh is generally chosen structured regular (even Cartesian if it is possible). Several local multi-grid methods have been developed, which differ in terms of the restriction operator used to correct the next coarser solution: the Local Defect Correction (LDC) method [31], the Flux Interface Correction (FIC) method [32] and the Fast Adaptive Composite (FAC) method [33].
The most obvious advantage of the local multi-grid approach over the standard multi-grid one is the gain in terms of CPU time and memory space when processing local singularity problems. However, as with the standard multi-grid approach, an iterative process has to be performed in order to obtain a converged solution at each level. The accuracy of the solution is therefore strongly dependent on the precision of the projection operators.

The performances of all these refinement methods depend strictly on the possibility of accurately detecting the zone of interest, i.e. the zone where the maximum discretisation error is present. To automate the refinement process, these methods are therefore often combined with *a posteriori* error estimators (e.g. [7,3,16,22,34,35]).

One category of error estimators, in which comparisons are made between two different grid refinements (e.g. [36,37,35]), are based on the principle that the discretisation error converges with the mesh step. These tools can be applied to physical situations of all kind.

Most of the *a posteriori* error estimators which are specifically dedicated to solid mechanics problems (e.g. [38–40]) are based on the fact that the classical finite element (FE) solution does not locally satisfy the static admissibility equation. The greater part of these estimators has been developed and proved in the context of linear behaviours. However, they can usually be extended to nonlinear behaviours (e.g. [34,41]), but are often unsuitable for dealing with the most complex situations (plasticity, large deformations, contact, friction, etc.). Further details about these estimators can be found in [42].

In this paper, the test cases studied were based on industrial problems including local singularities with different characteristic length-scales (see Section 2.1). The main constraint of this work was to develop a refinement strategy that can be easily implemented in any existing industrial software. In this context, we decided to perform a method relevant in a "black-box" solver context. Neither the local h-refinement method nor the p-refinement or the s-refinement strategies meet this criterion. In addition, we decided to use local multi-grid methods so that the memory space required for each resolution would not constitute an obstacle. This also enabled us to benefit from the excellent solver performances obtained on structured regular meshes. Among the existing local multi-level methods, the LDC method [31] was selected for this study because it seemed to be the most suitable method for dealing with solid mechanics problems involving local singularities.

Although the local multi-grid concept has been widely applied in the fields of thermodynamics and thermal hydraulics (e.g. [43–46]), only a few recent studies have focused on this approach in the context of solid mechanics [35,47]. The method presented here is fairly similar to that described in [35] but we proposed to use an existing convergence proved local multi-grid method [27,48,49], whereas Biotteau et al. adapted the Full Multi-Grid method [50] to deal with local refinement problems. The iterative processes used therefore differ in terms of the prolongation operators as well as the resolution steps.

This paper is organised as follows. In Section 2, the industrial test case studied and the problems involved are presented. The local defect correction (LDC) multi-level method is then presented in Section 3. Section 4 describes some strategies used to obtain accurate results with the LDC method. The accuracy, mesh convergence and automatic refinement procedures are focused on by performing 2-D and 3-D simulations. Lastly, Section 5 focuses on the performances of the LDC method with a view to applying it in industrial contexts, especially by making some comparisons with the existing global h-refinement method.

## 2. Context of the study

### 2.1. The industrial test case

The pellet-cladding interaction (PCI) [51] occurs during irradiation in pressurised water reactors, which constitute the majority of French nuclear reactors. The fuel consists in these reactors of cylindrical uranium dioxide ($UO_2$) pellets 8.2 mm in diameter and 13.5 mm high, which are piled up in a zircaloy cladding. During irradiation, the following two processes lead to the occurrence of PCI:

- The fuel pellets quickly crack during the first power increase (see Fig. 3, left). In addition, the fuel pellets swell and the cladding creeps due to the external pressure, which results in the occurrence of contacts between the pellets and the cladding. The pellet cracking process therefore results in discontinuous contacts.
- The deforming effects of the high temperature gradient on the pellets give them an hourglass shape (cf. Fig. 1). Contacts between the fuel and the pellets therefore occur first in front of the inter-pellet plane. High stress concentrations then develop all around the inter-pellet plane.

The localised stress concentrations combined with the discontinuous contacts with the pellets can lead to the failure of the cladding. It is therefore of great importance to be able to model PCI accurately because the integrity of the cladding, which is the first confinement barrier for the irradiated fuel, is at stake. Considerable research and development efforts are being made on this topic worldwide in order to understand the mechanisms possibly leading to PCI failure, as well as to design PCI resistant rods. It is difficult to perform complete 3-D simulations because too many DoF would be required to be able to model the local processes accurately. The strategy often used at present consists in using a structured conforming mesh locally refined around the PCI zone. The

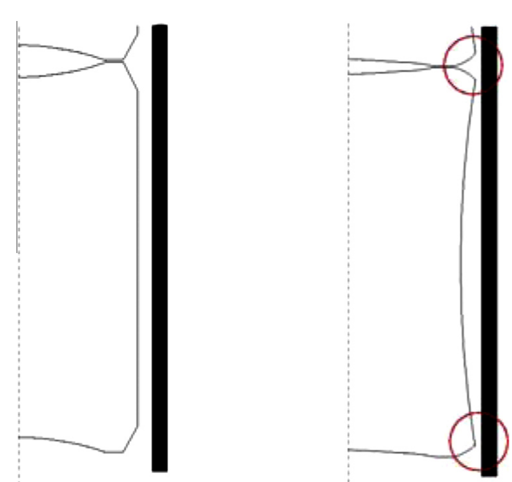

**Fig. 1.** Illustration of the hourglass shape deformation process: before (left) and during (right) irradiation.