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A three-dimensional elastoplastic constitutive model for concrete

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ABSTRACT

The nonlinear unified strength criterion proposed by authors in 2010 is adopted as the yield function and the plastic potential function for concrete. Four original material parameters of the nonlinear unified strength criterion are derived as functions of four conventional strength indexes of concrete. A three-dimensional elastoplastic constitutive model for concrete is then developed by using hardening and softening functions that are determined from the uniaxial compressive stress–strain relationship. The performance of the constitutive model is evaluated by comparing prediction and experimental data from literatures. The comparison shows the model is able to reasonably describe the three-dimensional strength characteristics, strain softening behavior after the peak stress and the dilatancy of various concretes. Moreover, the proposed model is integrated in the general finite element package ABAQUS via UMAT to simulate the deformation process of reinforced concrete columns under eccentric compression. The simulations show that the proposed constitutive model is able to describe the nonlinear mechanical behavior under complex stress states with high computational efficiency.

1. Introduction

The effects of hydrostatic pressure and intermediate principal stress on the strength of concrete [27,25,52,28,54,61], strain softening and dilatancy are basic mechanics behavior of concrete materials under complex stress state [8]. A constitutive model is a mathematical theory that describes the stress–strain relationships of materials, and reflects the deformation and strength characteristics of the material under an arbitrary stress state. In general, the constitutive models for concrete are formulated within the framework of a theory, such as continuum damage theory, elastoplasticity theory, and the combination of elastoplasticity and damage theories. Continuum damage theory is suitable for capturing the stiffness degradation of concrete, in which micro-cracking is considered through the variations of the elastic module. However, this theory fails to reproduce some important observed phenomena in concrete, such as irreversible deformations and dilatancy [36,12,34,11,62]. Elastoplasticity theory offers a more comprehensive modeling framework for concrete because it is able to describe the dilatancy, permanent strain and hardening/softening behavior of the concrete [43,20,50,33,29,44,16,48,32]. Combinations of elastoplasticity and damage theories are applied to describe the stiffness degradation through a damage parameter, the parameter is typically a function of the plastic strain of concrete. Namely, the influence of the existing plastic strain on the development of elastic strain is considered in the plastic-damage model by introducing the damage parameter. This type of constitutive model is often used to describe both the stiffness degradation and plastic strain accumulation under cyclic loading [37,2,57,41,1,58,18].

This paper focuses on the elastoplasticity constitutive model for concrete. This type of constitutive model is formulated by combining a strength criterion with the hardening and softening functions. The strength criteria are often adopted as the yield function that should be able to reflect the effect of hydrostatic pressure and intermediate principal stress on the deformation. In addition, they are also adopted as the plastic potential function having the capacity of governing the dilatancy of concrete reasonably. The most widely used strength criteria include the Willam–Warnke criterion [56], Ottosen criterion [42], Zienkiewicz–Pande criterion [63], Hsieh criterion [23], Menétrey–Willam criterion [40], and Pivonka criterion [47]. The hardening and softening functions are closely related to the hardening/softening parameter which are usually defined by the plastic volumetric strain, equivalent plastic shear strain or plastic work. And the hardening and softening functions are commonly obtained by the uniaxial compressive stress–strain relations. These stress–strain relationships are generally formulated by polynomial [22], rational [51] and piecewise functions [19,13]. A variety of

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three-dimensional constitutive models have been proposed based on elastoplasticity theory to reproduce the deformation and strength characteristics of concrete under multiaxial stress states. For instance, Imran and Pantazopoulou [26] proposed a constitutive model for concrete. In this model, non-associated plastic flow law was used, the Hsieh criterion (1988) was developed as the yield function, the Drucker–Prager-type criterion was selected as the plastic potential function, and a piecewise uniaxial compressive stress–strain function was used as the hardening and softening functions. The proposed constitutive model can only reflect the nonlinear deformation and strength characteristics of concrete under triaxial compressive conditions. By combining a hardening law using plastic volumetric strain as the hardening parameter with a yield function proposed by 40, Grassl et al. [17] established an elastoplastic constitutive model for concrete based on a non-associated flow rule. The constitutive model was reasonably able to describe the strain hardening and softening behavior of concrete under triaxial compressive conditions. Papanikolaou and Kappos [44] developed a confinement-sensitive plasticity constitutive model for concrete by combining the hardening law proposed by Grassl et al. [17] with the Zienkiewicz–Pande method. Park and Kim [45] proposed a three-yield-surface constitutive model that allows the behavioral characteristics of concrete in various stress states to be described accurately. Chi et al. [10] developed an elastoplasticity constitutive model for hybrid fiber-reinforced concrete, which comprises a five-parameter strength criterion proposed by Willam and Warnke [56], an uncoupled isotropic hardening/softening parameter determined by the accumulated equivalent plastic shear strain and a non-associated flow rule. Poltronieri et al. [48] proposed a simple elastoplastic constitutive model for concrete based on a recently proposed yield surface [5] and a class of hardening and softening rules that is a function of the accumulated plastic strain. These elastoplastic constitutive models employed the single strength criterion. Thus, they are suitable for a certain kind of concrete material. In addition, the parameters describing the effects of hydrostatic pressure and intermediate principal stress are typically coupled with each other. In practice, calibrating these parameters by conventional laboratory tests is extremely difficult.

To uniformly describe the three-dimensional strength characteristics of various types of concretes under complex stress conditions, Zienkiewicz and Pande [63] proposed a method in which a series of shape functions were used to replace the various strength criteria in the deviatoric plane, approximately. Many scholars established three-dimensional elastoplastic constitutive models based on the Zienkiewicz–Pande method, such as Brocca et al. [6], Grassl et al. [17], Aubertin and Li [3], Papanikolaou and Kappos [44], Lee et al. [30] and Tue et al. [55]. However, when these shape functions were used as yield functions of the elastoplastic constitutive model, they cannot reflect the stress-induced anisotropy for concrete [60]. Lu et al. [35] and Du et al. [14] proposed a nonlinear unified strength criterion that only requires four independent material parameters with clear physical meaning, and it was capable of describing the strength behavior of various types of concrete. Moreover, the strength surface was continuous and smooth in the principal stress space, and had a continuous partial derivative everywhere. Thus, the numerical calculations would easily lead to convergent solutions when the strength criterion was used as a yield function to establish the elastoplastic constitutive model for concrete. This paper employed the nonlinear unified strength criterion [35,14] as the yield function and plastic potential function and adopted the non-associated flow law to establish the elastoplastic constitutive model for concrete. Furthermore, new hardening and softening functions were acquired by using the uniaxial compressive stress–strain relation. A comparison of the experimental results of normal and high-strength concrete indicated that the deformation and strength characteristics of various types of concrete can be reasonably described by the constitutive model. The constitutive model was then integrated in the user-defined material mechanical behavior (UMAT) of the ABAQUS finite element program. The program simulated the results of experiments on the deformation process of a reinforced concrete column under eccentric compression that were conducted by the authors. The simulations proved that the constitutive models proposed in this paper are capable of reflecting the deformation rules of concrete under complex stress conditions with relatively high computation efficiency.

2. Material parameters of the nonlinear unified strength criterion

Material failure can be attributed to the deviatoric failure that occurs when the deviatoric stress on the failure plane reaches the shear strength. The Drucker–Prager criterion assumes that material failure occurs when the deviatoric stress in the octahedral plane reaches a critical value, and the deviatoric strength is a linear function of the normal stress. The Drucker–Prager criterion forms the upper bound of the nonlinear strength criteria. The Matsuoka–Nakai criterion [39] assumes that the deviatoric strength is a linear function of the normal stress on the spatially mobilized plane (SMP). The Matsuoka–Nakai criterion forms the lower bound of the nonlinear strength criteria. And all the nonlinear strength criteria ranging from the Matsuoka–Nakai criterion (lower bound) to the Drucker–Prager criterion (upper bound).

Based on the failure planes of the Drucker–Prager criterion and Matsuoka–Nakai criterion, Du et al. [14] used a material parameter to represent the position of the failure plane, which can be expressed as:

$$\vec{n} = \gamma\vec{n}_{\mathrm{M}} + (1-\gamma)\vec{n}_{\mathrm{S}} \tag{1}$$

where $\vec{n}_{\mathrm{M}}$ and $\vec{n}_{\mathrm{S}}$ are respectively the unit outer normal vector to the octahedral plane and SMP, and γ is a material parameter. As shown in Fig. 1, the failure plane changes from the SMP to the octahedral plane as γ changes from 0 to 1. The value of γ is different for different materials; and one material corresponds to a unique failure plane. The nonlinear unified strength criterion is applicable to a variety of materials, including concrete, rocks, soils, metals and other isotopic engineering materials.

The nonlinear unified strength criterion can be written as follow if the compressive stress is taken to be positive:

$$q_{\mathrm{s}}^{*} = \alpha\sqrt{\bar{I}_1^2 - 3\bar{I}_2} + \frac{2(1-\alpha)\bar{I}_1}{3\sqrt{(\bar{I}_1\bar{I}_2 - \bar{I}_3)/(\bar{I}_1\bar{I}_2 - 9\bar{I}_3)} - 1} = M_{\mathrm{f}}\bar{p} \tag{2}$$

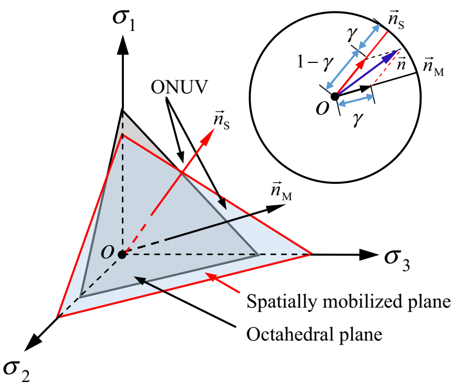

Fig. 1. Model of nonlinear unified strength criterion.