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Modeling interactions between political parties and electors

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HIGHLIGHTS

- We show how some operatorial tools can be used in the analysis of social systems.
- We propose a comparison between three different models, and we consider several explicit examples.
- We discuss the role of a "rule" acting on the system as a non-Hamiltonian term.

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ABSTRACT

In this paper we extend some recent results on an operatorial approach to the description of alliances between political parties interacting among themselves and with a basin of electors. In particular, we propose and compare three different models, deducing the dynamics of their related *decision functions*, i.e. the attitude of each party to form or not an alliance. In the first model the interactions between each party and their electors are considered. We show that these interactions drive the decision functions toward certain asymptotic values depending on the electors only: this is the *perfect party*, which behaves following the electors' suggestions. The second model is an extension of the first one in which we include a *rule* which modifies the status of the electors, and of the decision functions as a consequence, at some specific time step. In the third model we neglect the interactions with the electors while we consider cubic and quartic interactions between the parties and we show that we get (slightly oscillating) asymptotic values for the decision functions, close to their initial values. This is the *real party*, which does not listen to the electors. Several explicit situations are considered in details and numerical results are also shown.

1. Introduction

Mathematical modeling is a huge field of research which is applied to many different contexts, from physics to biology, from chemistry to finance. Tools and ideas usually adopted in physics have widely been used in these contexts. Pars pro toto, we cite [1], which is now considered a milestone in econophysics, [2–4], where the framework of statistical physics is used outside physics, and [5,6] as application of stochastic dynamics to social systems. Among all the strategies adopted during the years to build up models of some specific phenomenon, in recent years many researchers started to use methods

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typically connected with quantum mechanics, even when dealing with macroscopic systems. This has been done in decision-making processes, [7–12], in population dynamics, [13,14], in ecological processes, [15–17], and, recently, in the analysis of political systems, [18–22]. In these latter papers, the general operatorial settings analyzed in details in [12] have been used in the description of a *political system* consisting of three parties interacting among them and with a basin of electors and of undecided voters. The focus was on the so-called *decision function* (DF) of each party, i.e. on the attitude of the parties to form, or not, some alliance with the other parties. The main ingredient needed in the analysis of the time evolution of these DFs is a suitable Hamiltonian (i.e. a self-adjoint unbounded operator) which implements in itself all the mechanisms which are expected to take place in a realistic (but simplified) political system in which each party is treated as a single *variable* of the model. This is different from what was proposed in other mathematical models on the same subject, where the focus was on the single politician's behavior, see [23] and [24].

In this paper the crucial role in the procedure of decision-making of the parties is played by the various interactions existing between the parties and the basin of electors, which we treat as a kind of *open system* (the parties) interacting with a *reservoir* made of infinite degrees of freedom (the electors). This approach gives rise to several scenarios which are investigated all along the paper.

We also consider the possibility of implementing some effect which cannot be easily included in the Hamiltonian H of the system. In particular we adopt, for the first time in this context, a general procedure proposed first in [25], where the notion of (H,ρ)-induced dynamics was introduced. In particular, in [25], a *rule* ρ was added to H in the analysis of the quantum game of life: ρ is what is used to define the new state of the system at each iteration, because, for instance, the idea of the electors could be changed by what the parties are doing. Stated in different words, ρ is used to prepare the physical system for the iteration $k+1$ once the iteration k is performed. This enriches quite a lot the dynamics, and in fact several interesting results are deduced. In particular, we will discuss how a suitable rule, introduced to mimic the effect of the information *coming from the electors and reaching the parties*, can really deform the dynamics.

We will finally consider also the effect of nonlinear terms in the equations of motion. Something similar was done in [21], where the equations of motion were solved perturbatively, while here, paying the price of neglecting the interactions of the parties with their electors (i.e. taking the parameters measuring these interactions to be zero) and focusing only on the interactions among the parties, we are able to get analytical solutions for the DFs, improving significantly our previous results in this line.

The paper is organized as follows: in the next section we briefly discuss the model in [22], and we discuss what this model produces when the parameters are changed. Section 3 is devoted to the analysis of a deformed version of the previous model, deformation induced by the presence of a rule ρ. In Section 5 we propose a non quadratic Hamiltonian producing nonlinear, but still exactly solvable, equations of motion. Section 5 contains our conclusions. To keep the paper self-consistent, the Appendix contains some essential facts on the (H,ρ)-induced dynamics.

2. The first model

In this section, following [20,22], we consider a *physical system* $\mathcal{S}$ consisting, first of all, of three parties, $\mathcal{P}_1$, $\mathcal{P}_2$ and $\mathcal{P}_3$, which, together, form what we call $\mathcal{S}_\mathcal{P}$. Each party has to make a choice, and it can only choose one or zero, corresponding respectively to *form a coalition* with some other party or not. Hence we have $2^3=8$ different possibilities, which we associate to eight different and mutually orthogonal vectors in an eight-dimensional Hilbert space $\mathcal{H}_\mathcal{P}$. These vectors are called $\varphi_{i,k,l}$, with $i,k,l=0,1$. The three subscripts refer to whether or not the three parties of the model want to form a coalition at time $t=0$. Hence, for example, the vector $\varphi_{0,0,0}$, describes the fact that, at $t=0$, no party wants to ally with the other parties. Of course, this attitude can change during the time evolution, and deducing these changes is, in fact, what is interesting for us. This will be achieved, see below, by considering the mean values of some particular operators on these vectors or on some of their linear combinations. The set $\mathcal{F}_\varphi=\{\varphi_{i,k,l},\ i,k,l=0,1\}$ is an orthonormal basis for $\mathcal{H}_\mathcal{P}$. In general, a vector $\Psi_0=\sum_{i,k,l}\alpha_{i,k,l}\varphi_{i,k,l}$, with $\sum_{i,k,l}|\alpha_{i,k,l}|^2=1$, can be interpreted as a vector on $\mathcal{S}_\mathcal{P}$ in which the probability of finding $\mathcal{S}_\mathcal{P}$ in a state $\varphi_{i,k,l}$, at $t=0$, is given by $|\alpha_{i,k,l}|^2$. In particular, if for instance $\Psi_0=\varphi_{0,1,0}$, then the probability that, at $t=0$, $\mathcal{P}_1$ and $\mathcal{P}_3$ do not want to form any alliance while $\mathcal{P}_2$ does, is equal to one.

As it is shown in [20], it is convenient to construct the vectors $\varphi_{i,k,l}$ in a very special way, starting with the vacuum of three fermionic operators, p_1, p_2 and p_3, i.e. three operators which, together with their adjoints $p_1^\dagger$, $p_2^\dagger$ and $p_3^\dagger$, satisfy the canonical anticommutation relations (CAR) $\{p_k,p_l^\dagger\}=\delta_{k,l}$ and $\{p_k,p_l\}=0$. Then, $\varphi_{0,0,0}$ is a vector satisfying $p_j\varphi_{0,0,0}=0$, $j=1,2,3$, and the other vectors $\varphi_{i,k,l}$ can be constructed out of $\varphi_{0,0,0}$ as follows:

$$\varphi_{1,0,0}=p_1^\dagger\varphi_{0,0,0},\quad \varphi_{0,1,0}=p_2^\dagger\varphi_{0,0,0},\quad \varphi_{1,1,0}=p_1^\dagger p_2^\dagger\varphi_{0,0,0},\quad \varphi_{1,1,1}=p_1^\dagger p_2^\dagger p_3^\dagger\varphi_{0,0,0},$$

and so on. Let now $\hat{P}_j=p_j^\dagger p_j$ be the so-called *number operator* of the jth party. This operator satisfies $\hat{P}_j\varphi_{n_1,n_2,n_3}=n_j\varphi_{n_1,n_2,n_3}$, for $j=1,2,3$, and the eigenvalues n_j of these operators, zero and one, correspond to the only possible choices of the three parties at $t=0$, at least when the state of the system at $t=0$ is one of the vectors φ_{n_1,n_2,n_3}. More in general, if the initial state of the system is given by the vector Ψ_0 above, then

$$\hat{P}_1\Psi_0=\sum_{k,l}\alpha_{1,k,l}\varphi_{1,k,l},\quad \hat{P}_2\Psi_0=\sum_{i,l}\alpha_{i,1,l}\varphi_{i,1,l},\quad \hat{P}_3\Psi_0=\sum_{i,k}\alpha_{i,k,1}\varphi_{i,k,1},$$