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Modeling of the financial market using the two-dimensional anisotropic Ising model

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HIGHLIGHTS

- The two-dimensional Ising model in an external field and an ion single anisotropy term has been used as a mathematical model for the price dynamics of the financial market.
- The free energy of the model and the mean price $\langle S \rangle$ have been calculated using a mean field approximation.
- The influence of the anisotropy Δ on the behavior of the mean price has been gotten.

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ABSTRACT

We have used the two-dimensional classical anisotropic Ising model in an external field and with an ion single anisotropy term as a mathematical model for the price dynamics of the financial market. The model presented allows us to test within the same framework the comparative explanatory power of rational agents versus irrational agents with respect to the facts of financial markets. We have obtained the mean price in terms of the strong of the site anisotropy term Δ which reinforces the sensitivity of the agent's sentiment to external news.

1. Introduction

The study of complex systems in a unified framework has become recognized in recent years as a new scientific discipline, the ultimate of interdisciplinary fields. Among many things, the dynamics of prices of derivative securities has been studied in the literature since 1960s, where the celebrated Black Scholes formula [1] for the price of a European option was one of the first fundamental results in this direction. The development of the derivatives pricing theory has resulted in that, nowadays the volume of derivatives traded is much higher than the volume of basic assets [2].

Is well known that the modeling of a financial system with a large number of decision makers is analogous to modeling a physical system consisting of many degrees of freedom [3]. Since, the economical and sociological systems have been a great field for the application of concepts and mathematical methods of theoretical physics used to tackle complex systems [4–7]. One important model to treat the financial markets is the Ising model and its extensions [8–12]. This is a model many employed in statistical mechanics and which is very simple. It presents a binary variable S_i that makes it appealing to researchers from other branches of science including the economy [9]. However, there are different families of Econophysics models. For instance, the Mike and Farmer model [13] and the modified Mike and Former model [14]. Mike and Farmer have constructed a very powerful and realistic behavioral model to minimize the dynamic process of stock price formation based on the empirical regularities of order placement and cancellation in a purely order-driven market, which can successfully

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reproduce the whole distribution of returns, not only the well-known power-law tails, together with several other important stylized facts. The three key ingredients in the Mike–Farmer (MF) model: the long memory of order signs characterized by the Hurst index H_s, the distribution of relative order prices x in reference to the best price is described by a distribution such as the Student distribution and the dynamics of order cancellation. Gao-Feng Gu et al. [14] have improved the Mike–Farmer model for order-driven markets by introducing long memory in the order aggressiveness, which is a new stylized fact identified using the ultra-high-frequency data of 23 liquid Chinese stocks traded on the Shenzhen stock exchange in 2003.

From a general way, it is not much clear how to define the price of a market [9]. The only obvious requirement is that the price should go up when there is more demand than supply and vice versa. We can define the price x_i as a stochastic variable and as the normalized difference between demand. This behaves as the magnetization $x_t = 1/N\sum_{i=1}^{N} S_i(t)$ of a magnetic system, where N (the sites number) is the size of the system. Many studies have shown that the stock market fluctuations are inversely proportional to the frequency on some power that points to self-similarity in time for processes underlying the market [15,16,47].

One important thing in finance is the observation related to scaling laws in financial markets that is the widespread power-law behavior exhibited by large price changes. This is corroborated for practically all types of financial data and markets. The quantity of interest is the relative price change or return, defined as $r_t = (p_t - p_{t-1})/p_{t-1}$ where p_t denotes the price of an asset at time t [17–19]. The distribution of returns has been studied in detail and is well known present the inverse cubic-law $P(>v) \sim v^{-\beta}$, where $\beta \sim 3$ is the tail index and v is the volatility that can be defined as the modulus of the return, $v = |r|$ [14,20–26]. In contrast, empirical analyses for other stock markets have unveiled power-law tail exponents other than the Lévy regime and the inverse cubic-law. Makowiec and Gnaciński have studied the main index of Warsaw stock exchange in Poland for five years and found that the distribution of return follows power-law behaviors in three parts with $\beta = 0.76$, 2.03 and 3.88 for the positive tail and $\beta = 0.69$, 1.83 and 3.06 for the negative tail [27]. Bertram [28] focuses on the high-frequency dates of 200 most actively traded stocks in the Australian stock exchange in the period from 1993 to 2002, and reported that the distribution of returns has power-law tails with $\beta > 3$, which varies with different time interval Δt from 10 to 60 min. Coronel-Brizio et al. [29] analyzed the daily data (1990–2004) of the Mexican stock market index (IPC) and find that the distribution of the daily returns followed a power-law distribution with the exponent $\beta^+ = 3.33$ (positive tail) and $\beta^- = 3.12$ (negative tail) by selecting a suitable cutoff value [25]. Yan et al. [30] investigated the daily returns of 104 stocks (76 from the Shanghai stock exchange and 28 from Shenzhen stock exchange) in the Chines stock markets in the period from 1994 to 2001 and argue that the tail exponent is $\beta^+ = 2.44$ for the positive part and $\beta^- = 4.29$ for the negative part. After removing the opening and close returns of high-frequency data for the Shanghai stock exchange composite index, the tail exponents are much closer to $\beta = 3$. There are also controversial results for some markets. An example comes from the Indian stock market. Matia et al. [31] analyzed the daily returns of 49 largest stocks in the National stock exchange over 8 years (1994–2002) and find that the distribution of daily returns significantly deviates from the power-law form but decays exponentially in the form of $P(r) = e^{-\beta r}$ with the decay coefficient $\beta = 1.34$ for the positive tail and $\beta = 1.51$ for the negative tail. In contrast, Pan and Sinha [32] have studied the daily data of two stock indices (Nifty, 1990–2006 and Sensex, 1991–2006) and found the daily returns are exponentially distributed followed by power-law decay in the tails ($\beta^+ = 3.10$ and $\beta^- = 3.18$ for Nifty and $\beta^+ = 3.33$ and $\beta^- = 3.45$ for Sensex). They also analyze the high-frequency data of 489 stocks containing the information about all the transactions carried out in the national stock exchange (NSE) for two-year period (2003–2004) and observed power-law tails with $\alpha^+ = 2.87$ and $\alpha^- = 2.52$ for $\Delta t = 5$ and $\beta \sim 3$ for Δt ranging from 10 to 60 min.

The simple Ising spin model can be employed to describe the mechanism of price formation in financial markets. Its simplicity makes it appealing to researchers from other branches of science such as biology, sociology and economy [33–40]. In spite of simple rules, the model exhibits a complicated dynamics in one and more dimensions. In contrast to usual majority rules, in this model the influence was spreading outward from the center. This idea seemed appealing and we adapted it to model financial markets. New dynamic rules describing the behavior of two types of market players such as the trend followers and fundamentalists were obtained with the properties of simulated price trajectories duplicated those of analyzed historic data sets. Hence this simple and parameter free model is a good first approximation of a number of real financial markets [9,41].

In the present work we study the behavior of the mean price or return of the financial market using the Ising model in an external field with an anisotropy site term in the mean field approximation (MF). The plan of this paper is the following. In Section 2 we describe the model, in Section 3 we talk about the method employed and in Section 4, we present our conclusions and final remarks.

2. The model

The model is described by the following Hamiltonian

$$\mathcal{H} = \sum_{\langle ij \rangle} J_{ij} S_i S_j - B \sum_i S_i + \Delta \sum_i S_i^2 \tag{1}$$

where we make $J_{ij} = 1$, if i and j are neighboring sites, and $J_ij = 0$, otherwise. J_{ij} is interpreted here as the coefficient of influence of the agent j on agent i according to the following rule [8]: $J_{ij} = b_i + \alpha J_i(t-1) + \beta r(t-1)B(t-1)$, where b_i is