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Comparison of transfer entropy methods for financial time series

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HIGHLIGHTS

- We make a comparison between transfer entropy methods for financial time series.
- We introduce ERTE, which combines the advantages of both ETE and RTE.
- We analyse information flow and get reasonable results under different conditions.

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ABSTRACT

There is a certain relationship between the global financial markets, which creates an interactive network of global finance. Transfer entropy, a measurement for information transfer, offered a good way to analyse the relationship. In this paper, we analysed the relationship between 9 stock indices from the U.S., Europe and China (from 1995 to 2015) by using transfer entropy (TE), effective transfer entropy (ETE), Rényi transfer entropy (RTE) and effective Rényi transfer entropy (ERTE). We compared the four methods in the sense of the effectiveness for identification of the relationship between stock markets. In this paper, two kinds of information flows are given. One reveals that the U.S. took the leading position when in terms of lagged-current cases, but when it comes to the same date, China is the most influential. And ERTE could provide superior results.

1. Introduction

With the development of economic society, the trend towards economic globalization has grown stronger. Stocks, regarded as economic indicator, have become the most intuitive criterion for the public to judge the economic situation at the moment.

Transfer entropy, proposed by the German scholar Schreiber in 2000 [1], has received wide attention since it was put forward. It derived from Shannon entropy [2]. Compared to the mutual information entropy [3] which is a measurement of the statistical correlation of two random variables for relationship, transfer entropy described the direction of information flow in a more explicit way. Because of this characteristic, it has been applied in a variety of areas in research. Lizier [4] introduced transfer entropy (TE) into cellular automata and demonstrated to be a method of filtering for coherent structures. Sumioka [5], Papana [6] and Faes [7] applied this method to physiological time series in order to detect

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causality between interactions on neural level. Ver Steeg [8] summarized this methods for social media. In the field of financial time series, Michael Lindner et al., [9] proposed a Matlab toolbox, named Trentool, to solve financial problems with transfer entropy. Kang [10] applied transfer entropy to financial time series in order to analyse the relationship between stock indices from the US and China, and she gave the direction between indices which was coincident with the fact. Leonidas Sandoval [11]constructed a financial network framework based on TE to analyse the contagion of the stock markets and reduce risk in the financial crisis. There comes two extended concepts about TE, the one is effective transfer entropy (ETE) [12], the other is Rényi transfer entropy (RTE) [13]. ETE offered an improved method of TE that it can weaken the influence aroused by noise. And RTE, derived from Rényi entropy, provided a way to calculate TE by adjusting parameter q, which could describe the effect of different probability intervals on the results. Huang et al., [14] applied Rényi entropy to focus on the scaling for regular and extreme volatility in developed and emerging markets, and they successfully distinguished their differences where regular volatility exhibited a long-range persistence while extreme volatility expressed anti-persistence. Dimpfl [15] applied RTE to analyse high frequency financial data. Beyond that, another widely used method that is similar to TE is Granger causality [16]. That is, in the case of time series with two economic variables X, Y, the prediction in the condition of containing both the past information of X and Y is better than that only based on itself.

In summary, TE is an effective way to measure information transfer, especially in nonlinear systems, and it can effectively distinguish the driving elements from responding ones. In this paper, we applied this method to financial time series in order to analyse the causal relationship between stocks, which is a good way to make clear the information transfer between international financial markets. And we introduced three approaches for a comparative study, from which we could know the advantages and disadvantages of each method.

In this paper, we analysed the relationship among financial time series based on a series of transfer entropy methods and the results are compared. We chose data from 9 stocks during 1995 and 2015. Owing to the selection of the time period, which contained the financial crisis in 2008, we analysed the data from 2008 separately. In order to improve the performance of method, we introduced a modified transfer entropy based on ETE and RTE, called effective Rényi transfer entropy (ERTE). We used the shuffle algorithm of ETE into ERTE to reduce the noise.

The rest of this paper was organized as follows. In Section 2, the definition of transfer entropy, effective transfer entropy, Rényi transfer entropy and effective Rényi transfer entropy were presented, and data shuffling algorithm were introduced. The data simulation was delivered in Section 3. Section 4 demonstrated the results of those methods. Conclusion was in Section 5.

2. Methods

2.1. Shannon entropy

Information theory was firstly proposed by Shannon in 1948. The article, named "Mathematical theory of communication", is regarded as the sign of information theory's birth. And the author of the article, Shannon, is considered as the founder of information theory. This theory takes probability theory into analyse the information of communication. After decades of development, it has been gradually improved and widely used in the fields of computer science, biology, statistics, sociology, economics etc. In general, the information theory we mention often refer to the classical information theory, that is, Shannon information theory.

According to the definition of Shannon information theory, information means the decrease of uncertainty. So, we can measure information by calculating the decrease of uncertainty.

The definition of Shannon entropy is

$$H(X) = H(p_1, p_2, \ldots, p_K) = \sum_{k=1}^{K} p_k I(x_k) = \sum_k p_k \log \frac{1}{P(x_k)} = -\sum_k p_k \log P(x_k) \tag{1}$$

where $P(x_k)$ is the prior probability of x_k.

When it comes to the 'relationship' between two different sequences, generally, people always describe it with mutual information. Mutual information is a measurement of information in the information theory, it can be considered as the information from a random variable which contains information about another.

Assuming that the joint distribution of two random variables (X, Y) is $p(x, y)$, two marginal distributions are $p(x)$ and $p(y)$, thus, mutual information $I(X; Y)$ is the relative entropy of the joint distribution $p(x, y)$ and the product distribution $p(x)p(y)$. Or we can describe it like this, according to chain rule of entropy, there is

$$H(X, Y) = H(X) + H(Y|X) = H(Y) + H(X|Y). \tag{2}$$

Thus, according to the definition of mutual entropy, we have

$$\begin{aligned} I(X; Y) &= H(X) - H(X|Y) \\ &= H(X) + H(Y) - H(X, Y) \end{aligned}$$