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Transport coefficients for relativistic gas mixtures of hard-sphere particles

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HIGHLIGHTS

- Fourier, Fick and Navier–Stokes laws for relativistic mixtures.
- Transport coefficients for hard-sphere potential.
- Dependence of the transport coefficients on the gravitational field.
- Relativistic mixtures in gravitational fields.

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ABSTRACT

In the present work, we calculate the transport coefficients for a relativistic binary mixture of diluted gases of hard-sphere particles. The gas mixture under consideration is studied within the relativistic Boltzmann equation in the presence of a gravitational field described by the isotropic Schwarzschild metric. We obtain the linear constitutive equations for the thermodynamic fluxes. The driving forces for the fluxes of particles and heat will appear with terms proportional to the gradient of gravitational potential. We discuss the consequences of the gravitational dependence on the driving forces. We obtain general integral expressions for the transport coefficients and evaluate them by assuming a hard-sphere interaction amongst the particles when they collide and not very disparate masses and diameters of the particles of each species. The obtained results are expressed in terms of their temperature dependence through the relativistic parameter which gives the ratio of the rest energy of the particles and the thermal energy of the gas mixture. Plots are given to analyze the behavior of the transport coefficients with respect to the temperature when small variations in masses and diameters of the particles of the species are present. We also analyze for each coefficient the corresponding limits to a single gas so the non-relativistic and ultra-relativistic limiting cases are recovered as well. Furthermore, we show that the transport coefficients have a dependence on the gravitational field.

1. Introduction

An inspection of the literature (see e.g. the Refs. [1–12]) concerning the analysis of relativistic gas mixtures shows that in the majority of the works, only general expressions for the transport coefficients have been given. One interesting feature of the transport coefficients of relativistic gases is their dependence on a parameter $\zeta_a = m_a c^2/kT$ which gives the ratio of the rest energy of the particles of species a and the thermal energy of the gas. This ratio is small for high temperatures so

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the gas is in the ultra-relativistic regime, for low temperatures such a ratio is large and then the gas is in the non-relativistic regime. For a single gas the expressions for the transport coefficients in these limits are well known (see e.g. [2,11,12]). For mixtures of relativistic gases there are only few works which analyze the transport coefficients. We quote: Ref. [13] where the reaction rate coefficient was obtained by considering a relativistic reactive differential cross-section which takes into account the activation energy of the chemical reaction; Ref. [14] where Grad's moment method was employed for the determination of the transport coefficients for mixtures of Maxwellian particles; lastly Ref. [15] where the diffusion coefficient was determined by using a BGK-type kinetic model for a mixture of hard-sphere particles.

Some researches of relativistic gases in the presence of gravitational fields have been recently performed, we quote for instance the works [15–19]. Among the results, it is interesting to note that relativistic effects arise in the form of the thermal and diffusion generalized forces. Such forces have a contribution due to the four-acceleration as originally presented by Eckart [20] for the thermal force and a gravitational potential gradient in accordance with Tolman's law [21,22].

This report focuses the evaluation of the transport coefficients for a binary mixture of relativistic ideal gases and represents a continuation of the research presented in Ref. [19]. In such a reference we used a method of solution for the Boltzmann equation that combines the Chapman–Enskog and Grad representations (see e.g. [12,23]). In Ref. [19] we found the linear constitutive equations for the heat and particle fluxes, dynamic pressure and pressure deviator tensor, and gave the transport coefficients as general integrals for a gas mixture. In the present paper we rewrite such expressions in the particular case of a binary mixture and evaluate them by assuming two physical hypotheses: (1) a hard-sphere interaction of the particles when they collide and (2) not very disparate molecular masses and diameters for the species. The performance of the integrals for the determination of the transport coefficients represents long and tricky manipulations. For this reason we have added in the Appendix A sufficient hints to reproduce all the calculations. In Appendix B we have listed a number of integrals that appear along the process taking into account the two physical hypotheses mentioned before. We present the expressions for the transport coefficients when they depend on differences of masses and diameters of the particles, and on concentrations of the species. We explore their behavior by analyzing some graphics with respect to the relativistic parameter ζ_a. We analyze the one-species limits and we show that they are in accordance with those reported in the literature [24,25,23]. Of course this last statement guarantees the correct ultra-relativistic (high temperatures) and non-relativistic (low temperatures) limits for each transport coefficient. We also show that, due to the presence of the gravitational field, the transport coefficients become smaller.

Applications of these results can be done to some astrophysical situations like white dwarfs or clouds nearby a source of gravitational potential. An example of self-diffusion can be addressed to a situation in which different isotopes of the same gas diffuse through each other; in this case the mass difference between the components of the gas can be very small.

The structure of the work is as follows. In Section 2 we recall the basic equations and definitions. In Section 3 we analyze the shear and bulk viscosity coefficients for a binary mixture and the limiting cases for a simple fluid are given. Section 4 is devoted to the determination of the thermal conductivity, diffusion and thermal-diffusion rate coefficients; there the one-component limits are also given. The influence of the gravitational field on the transport coefficients is presented in Section 5 and finally the main conclusions are stated in Section 6.

2. Background

In this work we deal with a binary mixture of diluted ideal gases in the nearby of a gravitational potential produced by a spherical static source. We assume a curved space–time described with the isotropic Schwarzschild metric $g^{\mu\nu}$:

$$ds^2 = g_0(r)\left(dx^0\right)^2 - g_1(r)\delta_{ij}dx^i dx^j, \quad g_0(r) = \left(\frac{1+\frac{\Phi}{2c^2}}{1-\frac{\Phi}{2c^2}}\right)^2, \; g_1(r) = \left(1-\frac{\Phi}{2c^2}\right)^4. \tag{1}$$

The above equations contain the gravitational potential $\Phi = -GM/r$, where G is the gravitational constant, M the total mass of the spherical source, r the corresponding radius and c the light speed and indexes $\{i, j\}$ will run over spatial components.

The gases under consideration are composed by particles that do not have internal degrees of freedom and are characterized by their space–time coordinates $x^\mu = (ct, \mathbf{x})$. The four-momentum of each particle is $p_a^\mu = \left(p_a^0, \mathbf{p}_a\right)$ where the Latin subindex $a = 1, 2$ denotes the species of the gas whereas the Greek index μ denotes the tensor properties. The momentum of each particle represents a time-like four-vector that holds the so-called mass-shell condition, i.e., $g_{\mu\nu}p_a^\mu p_a^\nu = m_a^2c^2$, where m_a is the rest mass of a particle of species a. This last condition leads to a relation between the temporal and spatial components of the four-momentum in the following way:

$$p_a^0 = p_{a0}/g_0, \quad \text{and} \quad p_{a0} = \sqrt{g_0\left(m_a^2c^2 - g_1|\mathbf{p}_a|^2\right)} \tag{2}$$

for the contravariant and covariant representations, respectively.

Statistical mechanics has as a basis the macroscopic description of a gas through a distribution function. As usual we will use the one-particle distribution function $f_a\left(x^\mu, p_a^\mu\right)$ so that $f_a\left(x^\mu, p_a^\mu\right) d^3x\, d^3p_a$ is the number of particles of the constituent a in the volume element between $\mathbf{x}$, $\mathbf{x} + d^3x$ and $\mathbf{p}_a$, $\mathbf{p}_a + d^3p_a$ at some instant of time t. This last quantity f_a can be obtained