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On triply coupled vibrations of axially loaded thin-walled composite beams

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ABSTRACT

Free vibration of axially loaded thin-walled composite beams with arbitrary lay-ups is presented. This model is based on the classical lamination theory, and accounts for all the structural coupling coming from material anisotropy. Equations of motion for flexural–torsional coupled vibration are derived from the Hamilton's principle. The resulting coupling is referred to as triply coupled vibrations. A displacement-based one-dimensional finite element model is developed to solve the problem. Numerical results are obtained for thin-walled composite beams to investigate the effects of axial force, fiber orientation and modulus ratio on the natural frequencies, load–frequency interaction curves and corresponding vibration mode shapes.

1. Introduction

Fiber-reinforced composite materials have been used over the past few decades in a variety of structures. Composites have many desirable characteristics, such as high ratio of stiffness and strength to weight, corrosion resistance and magnetic transparency. Thin-walled structural shapes made up of composite materials, which are usually produced by pultrusion, are being increasingly used in many engineering fields. However, the structural behavior is very complex due to coupling effects as well as warping-torsion and therefore, the accurate prediction of stability limit state and dynamic characteristics is of the fundamental importance in the design of composite structures.

The theory of thin-walled members made of isotropic materials was first developed by Vlasov [1] and Gjelsvik [2]. Up to the present, investigation into the stability and vibrational behavior of these members has received widespread attention and has been carried out extensively. Closed-form solution for the flexural and torsional natural frequencies, critical buckling loads of isotropic thin-walled bars are found in the literature (Timoshenko [3,4] and Trahair [5]). For some practical applications, earlier studies have shown that the effect of axial force on the natural frequencies and mode shapes is more pronounced than those of the shear deformation and rotary inertia. Many numerical techniques have been used to solve the dynamic analysis of thin-walled members. One of the most effective approach is to derive the exact stiffness matrices based on the solution of the governing differential equations of motion. Most of those studies adopted an analytical method that required explicit expressions of exact displacement functions for governing equations. Although a large number of studies have been performed on the dynamic characteristics of axially loaded isotropic thin-walled beams [6–9], it should be noted that by using this method there appear some works reported on the free vibration of axially loaded thin-walled closed-section composite beams (Banerjee et al. [10–12], Li et al. [13,14] and Kaya and Ozgumus [15]). For thin-walled open-section composite beams, the works of Kim et al. [16–18] deserved special attention because they evaluated not only the exact element stiffness matrix but also dynamic stiffness matrix to perform the spatially coupled stability and vibration analysis of thin-walled composite I-beam with arbitrary laminations. By using finite element method, Bank and Kao [19] analyzed free and forced vibration of thin-walled composite beams. Cortinez, Machado and Piovan [20,21] presented a theoretical model for the dynamic analysis of thin-walled composite beams with initial stresses. Machado et al. [22] determined the regions of dynamic instability of a simply supported thin-walled composite beam under an axial excitation. The analysis was based on a small strain and moderate rotation theory, which was formulated through the adoption of a second-order displacement field. In their research [20–22], thin-walled composite beams for both open and closed cross-sections and the shear flexibility (bending, non-uniform warping) were incorporated. However, it was strictly valid for symmetric balanced laminates and especially orthotropic laminates. By using a boundary element method, Sapountzakis and Tsiatas [23] solved the flexural–torsional buckling and vibration problems of Euler–Bernoulli composite beams with arbitrarily cross section. This method overcame the shortcoming of possible thin tube theory solution, which its utilization had been proven to be prohibitive even in thin-walled homogeneous sections.

In this paper, which is an extension of the authors' previous works [24–27], flexural–torsional coupled vibration of axially

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loaded thin-walled composite beams with arbitrary lay-ups is presented. This model is based on the classical lamination theory, and accounts for all the structural coupling coming from the material anisotropy. The governing differential equations of motion are derived from the Hamilton's principle. A displacement-based one-dimensional finite element model is developed to solve the problem. Numerical results are obtained for thin-walled composite beams to investigate the effects of axial force, fiber orientation and modulus ratio on the natural frequencies and load–frequency interaction curves as well as corresponding vibration mode shapes.

2. Kinematics

The theoretical developments presented in this paper require two sets of coordinate systems which are mutually interrelated. The first coordinate system is the orthogonal Cartesian coordinate system (x, y, z), for which the x and y axes lie in the plane of the cross section and the z axis parallel to the longitudinal axis of the beam. The second coordinate system is the local plate coordinate (n, s, z) as shown in Fig. 1, wherein the n axis is normal to the middle surface of a plate element, the s axis is tangent to the middle surface and is directed along the contour line of the cross section. The (n, s, z) and (x, y, z) coordinate systems are related through an angle of orientation θ. As defined in Fig. 1 a point P, called the pole, is placed at an arbitrary point x_p, y_p. A line through P parallel to the z axis is called the pole axis.

To derive the analytical model for a thin-walled composite beam, the following assumptions are made:

1. The contour of the thin wall does not deform in its own plane.
2. The linear shear strain $\bar{\gamma}_{sz}$ of the middle surface is zero in each element.
3. The Kirchhoff–Love assumption in classical plate theory remains valid for laminated composite thin-walled beams.
4. Each laminate is thin and perfectly bonded.
5. Local buckling is not considered.

According to assumption 1, the midsurface displacement components $\bar{u}, \bar{v}$ at a point A in the contour coordinate system can be expressed in terms of a displacements U, V of the pole P in the x, y directions, respectively, and the rotation angle Φ about the pole axis,

$$\bar{u}(s,z) = U(z)\sin\theta(s) - V(z)\cos\theta(s) - \Phi(z)q(s) \quad (1a)$$
$$\bar{v}(s,z) = U(z)\cos\theta(s) + V(z)\sin\theta(s) + \Phi(z)r(s) \quad (1b)$$

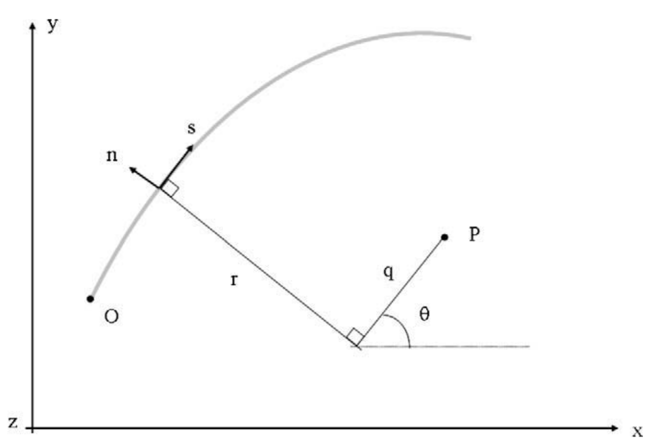

Fig. 1. Definition of coordinates in thin-walled open sections.

These equations apply to the whole contour. The out-of-plane shell displacement $\bar{w}$ can now be found from the assumption 2. For each element of middle surface, the shear strain become

$$\bar{\gamma}_{sz} = \frac{\partial \bar{v}}{\partial z} + \frac{\partial \bar{w}}{\partial s} = 0 \quad (2)$$

After substituting for $\bar{v}$ from Eq. (1) and considering the following geometric relations,

$$dx = ds\cos\theta \quad (3a)$$
$$dy = ds\sin\theta \quad (3b)$$

Eq. (2) can be integrated with respect to s from the origin to an arbitrary point on the contour,

$$\bar{w}(s,z) = W(z) - U'(z)x(s) - V'(z)y(s) - \Phi'(z)\omega(s) \quad (4)$$

where differentiation with respect to the axial coordinate z is denoted by primes ($'$); W represents the average axial displacement of the beam in the z direction; x and y are the coordinates of the contour in the (x, y, z) coordinate system; and ω is the so-called sectorial coordinate or warping function given by

$$\omega(s) = \int_{s_\circ}^{s} r(s)ds \quad (5a)$$

The displacement components u,v,w representing the deformation of any generic point on the profile section are given with respect to the midsurface displacements $\bar{u}, \bar{v}, \bar{w}$ by the assumption 3.

$$u(s,z,n) = \bar{u}(s,z) \quad (6a)$$
$$v(s,z,n) = \bar{v}(s,z) - n\frac{\partial \bar{u}(s,z)}{\partial s} \quad (6b)$$
$$w(s,z,n) = \bar{w}(s,z) - n\frac{\partial \bar{u}(s,z)}{\partial z} \quad (6c)$$

The strains associated with the small-displacement theory of elasticity are given by

$$\epsilon_s = \bar{\epsilon}_s + n\bar{\kappa}_s \quad (7a)$$
$$\epsilon_z = \bar{\epsilon}_z + n\bar{\kappa}_z \quad (7b)$$
$$\gamma_{sz} = \bar{\gamma}_{sz} + n\bar{\kappa}_{sz} \quad (7c)$$

where

$$\bar{\epsilon}_s = \frac{\partial \bar{v}}{\partial s}; \quad \bar{\epsilon}_z = \frac{\partial \bar{w}}{\partial z} \quad (8a)$$
$$\bar{\kappa}_s = -\frac{\partial^2 \bar{u}}{\partial z^2}; \quad \bar{\kappa}_z = -\frac{\partial^2 \bar{u}}{\partial z^2}; \quad \bar{\kappa}_{sz} = -2\frac{\partial^2 \bar{u}}{\partial s \partial z} \quad (8b)$$

All the other strains are identically zero. In Eq. (8), $\bar{\epsilon}_s$ and $\bar{\kappa}_s$ are assumed to be zero. $\bar{\epsilon}_z, \bar{\kappa}_z$ and $\bar{\kappa}_{sz}$ are midsurface axial strain and biaxial curvature of the shell, respectively. The above shell strains can be converted to beam strain components by substituting Eqs. (1), (4) and (6) into Eq. (8) as

$$\bar{\epsilon}_z = \epsilon_z^\circ + x\kappa_y + y\kappa_x + \omega\kappa_\omega \quad (9a)$$
$$\bar{\kappa}_z = \kappa_y\sin\theta - \kappa_x\cos\theta - \kappa_\omega q \quad (9b)$$
$$\bar{\kappa}_{sz} = 2\bar{\chi}_{sz} = \kappa_{sz} \quad (9c)$$

where $\epsilon_z^\circ, \kappa_x, \kappa_y, \kappa_\omega$ and κ_{sz} are axial strain, biaxial curvatures in the x and y direction, warping curvature with respect to the shear center, and twisting curvature in the beam, respectively defined as

$$\epsilon_z^\circ = W' \quad (10a)$$
$$\kappa_x = -V'' \quad (10b)$$
$$\kappa_y = -U'' \quad (10c)$$
$$\kappa_\omega = -\Phi'' \quad (10d)$$
$$\kappa_{sz} = 2\Phi' \quad (10e)$$