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Asynchronous updates can promote the evolution of cooperation on multiplex networks

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HIGHLIGHTS

- Cooperation increases on multiplex networks if strategies update asynchronously.
- This effect is largest when payoff differences between network nodes are large.
- The size of the increase in cooperation depends on the social dilemma.

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ABSTRACT

We study the importance to the frequency of cooperation of the choice of updating strategies in a game played asynchronously or synchronously across layers in a multiplex network. Updating asynchronously in the public goods game leads to higher frequencies of cooperation compared to synchronous updates. How large this effect is depends on the sensitivity of the game dynamics to changes in the number of cooperators surrounding a player, with the largest effect observed when players payoffs are small. The discovery of this effect enhances understanding of cooperation on multiplex networks, and demonstrates a new way to maintain cooperation in these systems.

1. Introduction

In many social systems a social dilemma exists where what is good for the individual is not necessarily good for the group, and so explaining why in both laboratory [1,2] and field [3,4] experiments participants are found to cooperate (act in the interests of the group) more frequently than would be expected for purely rational self-interested players has proved a challenge to evolutionary biology and sociology. The prevalence of cooperation between unrelated members of a population in both biological and human social systems has been the subject of a long history of study, with a number of key insights over the past few decades [5].

One of the central tools used to understand the resolution of social dilemmas and the evolution of cooperation is game theory [6]. Here each player can choose from a number of strategies with a payoff, dependent on their own strategy and those chosen by their opponents, indicating to each player how well they are performing. Playing the prisoner's dilemma (PD) on a lattice network (where the network defines those that each player plays against) cooperation has been shown to increase compared to the unstructured case [7] by cooperators forming clusters that are resistant to exploitation by surrounding

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defectors, an effect known as network reciprocity. Developing these ideas the evolution of cooperation on networks received intense subsequent study, with the effect on cooperation of network structure [8–11], the type of game [12], the rules used to update the strategy [13], additional strategies such as punishment and loners [14–16] and diversity between players [17] investigated. In each case it was found that added heterogeneity introduces higher levels of cooperation, as cooperators are able to seed clusters more easily and boost the effect of network reciprocity.

In scenarios such as electricity power networks and networks of air travel it is more accurate to describe a system as a combination of networks than a single one [18–22]. These multilayered systems can take two forms: either interdependent networks, where two separate networks are connected through a number of between-network edges, or multiplex networks, where the nodes on each layer represent the same entity, but each layer represents a different aspect of the system [23].

How the amount of cooperation on a multilayered system varies has been studied in a number of cases, from interdependent networks of different topologies [24,25] to lattices with weighted payoffs between the layers [26,27] and weighted fitnesses [28], probabilistic connections between layers [29,30], different imitation rules between layers [31] and memory [32]. In each of these cases cooperation is enhanced by the addition of extra networks caused by strategies on each layer supporting each other against exploitation by defectors, and there is a peak interdependence between the two layers due to maximum heterogeneity in the system, even when the games played on each network are different [33].

The dynamics of games have also been studied on multiplex networks [34]. These are networks with more than one layer, where the nodes on each layer represent the same player, but the connections between them are not necessarily the same. On these networks players can play different strategies on each layer, but their payoffs are summed across all layers. Cooperation was found to be increased compared to the single layer when games are played on random network multiplexes due to the ability of players to play a differing strategy on each layer [35,36]. These "incoherent" players are able to maintain cooperation when it would usually have disappeared in the single layer system. The effect of separating the networks on which players calculate their payoffs and those on which they update their strategies onto different layers has been explored [37], as has how the assortativity of the degrees between the networks affects the final frequency of cooperation, what happens if the game played on each layer of the multiplex is different [38], and if players imitate those that they are most similar to [39]. Zhang et al. [40] found the analytical conditions for cooperation on a multiplex by studying structured groups across a two-layer network, where cooperation was found to be maximised when intermediate levels of migration between groups was implemented.

Despite the existing work discussing cooperation in multilayered networks, more needs to be done in order to understand exactly what is important in these systems for cooperation to be maintained, and how cooperation can be encouraged in order to offer solutions to social dilemmas. One assumption that is made in a number of previous models is about how frequently the nodes on each layer update themselves. It has been found [13,41] that the proportion of nodes that update their strategies on a network before the payoff of each player is recalculated can alter the amount of cooperation in the system. What has not been considered in the case of multilayered networks is whether updating each of the nodes across all layers compared to just a single layer at a time affects the final amount of cooperation in the system. Whether a player updates their strategy on all layers of the system at the same time or separately will depend strongly on the system that is to be modelled. It is therefore important to know whether this has any effect on the final frequency of cooperation in the system. This is the question that we will address in this article.

2. Mathematical model

In order to study the spread of cooperation on the multiplex network the public goods game (PGG) is played by each of the nodes. In this game there are two possible strategies, either cooperation or defection. In the PGG the players are divided into groups before donating a certain amount to the groups that they are members of. The total that has been donated is then multiplied up by an enhancement factor, before being divided between all of the members of that group. When playing the PGG on a network the groups are defined to be those players that are connected to a common node. So, on each layer the player plays the game in $k + 1$ groups, where k is the degree of the player on that particular layer. In the PGG, player i donates an amount c_i, and so the payoffs are given by

$$P_D^i = r\frac{\sum_{j=1}^{N_C} c_j}{G} \tag{1}$$

$$P_C^i = r\frac{\sum_{j=1}^{N_C} c_j + c_i}{G} - c_i \tag{2}$$

where $P_{C,D}^i$ is the payoff of player i playing as a cooperator or a defector respectively, G is the number of players in the group, N_C is the number of cooperators in the group, r is the enhancement factor (the return on the investment to the group) and the last term in Eq. (2) represents the donation by the cooperators to the public good. The final payoff for each player is found by summing across all of the groups in which they play across all of the layers. We will study two possible versions of the PGG: fixed cost per group (FCG) when each player donates one unit to whichever group they are playing the game in