Physica A 479 (2017) 478–484

Contents lists available at ScienceDirect

Physica A

journal homepage: www.elsevier.com/locate/physa

A lattice hydrodynamic model based on delayed feedback control considering the effect of flow rate difference

Yunong Wang, Rongjun Cheng *, Hongxia Ge

Faculty of Maritime and Transportation, Ningbo University, Ningbo 315211, China
Jiangsu Key Laboratory of Urban ITS Southeast University, Nanjing 210096, China
National Traffic Management Engineering and Technology Research Centre Ningbo University Sub-centre, Ningbo 315211, China

HIGHLIGHTS

- A delayed feedback control is proposed based on the lattice hydrodynamic model.
- The effect of flow rate difference between sequential lattice sites is considered.
- Numerical simulation is given to prove the advantages of theoretical analyses.
- The traffic jams can be effectively suppressed by the control scheme.

ARTICLE INFO

Article history:
Received 18 October 2016
Received in revised form 27 December 2016
Available online 22 March 2017

Keywords:
Traffic flow
Lattice hydrodynamic model
Delayed feedback control
Stability condition

ABSTRACT

In this paper, a lattice hydrodynamic model is derived considering not only the effect of flow rate difference but also the delayed feedback control signal which including more comprehensive information. The control method is used to analyze the stability of the model. Furthermore, the critical condition for the linear steady traffic flow is deduced and the numerical simulation is carried out to investigate the advantage of the proposed model with and without the effect of flow rate difference and the control signal. The results are consistent with the theoretical analysis correspondingly.

1. Introduction

For several past decades, the traffic congestion has closely impacted on human's daily life, such as traffic accident, air pollution and global warming. So it has been highly recommended to attach importance to the traffic congestion problem with mathematical physics and control theories. Up to present, a large number of traffic flow models are proposed including the car-following models [1–17], the cellular automaton models [18–21], the hydrodynamic models [22–44], and the gas kinetic models [45,46] to figure out the complicated constitution behind the traffic congestion phenomenon.

In 1995, a famous car-following model called optimal velocity model (OVM, for short) was put forward by Bando et al. [7]. In OVM, vehicle's acceleration at the same time was determined by the difference between actual velocity and an optimal velocity. Based on OVM, a multitude of car-following models have been extended with delayed feedback control from the view of control theory [8–13]. In 2006, Zhao and Gao [14] proposed a simple coupled-map(CM) car-following model considering the velocity feedback control to suppress the traffic jam. Subsequently, Han [15] and Ge [16] took into account the ITS with CM car-following model in 2007 and 2011 respectively. Then in 2014, Li [17] presented a dynamic collaboration model with feedback signals to suppress the traffic congestion. Although the application of feedback control theory becomes

* Corresponding author at: Faculty of Maritime and Transportation, Ningbo University, Ningbo 315211, China.
E-mail address: chengrongjun@nbu.edu.cn (R. Cheng).

http://dx.doi.org/10.1016/j.physa.2017.03.027

more and more mature in the field of microscopic, there are seldom studies hammer at traffic jam problem for macroscopic models with control theory.

The lattice hydrodynamic model was firstly introduced by Nagatani [27,28] to summarize the variational relationships between collective variables. After that, the lattice hydrodynamic model was universally studied by considering different factors from the macroscopic viewpoint [29–41], but the investigation of control signal in lattice hydrodynamic models is rare. In 2015, Redhu [42] carried out the DFC method for lattice hydrodynamic model considering the flux change in adjacent time and Ge [43] presented a simple control method with the lattice hydrodynamic model by applying a decentralized delayed-feedback control. Recently, Li [44] studied the lattice hydrodynamic model performance based on delayed feedback control with a view of density change rate difference.

During the actual driving condition, the dramatic change of velocity and acceleration not only impacts on the whole traffic flow, but also steps up the vehicle emissions, and we call it the jerk profile [47]. As we know, jerk is the derivative of acceleration, analogously we apply it into lattice hydrodynamic model to describe the flow rate difference. But up to now, no studies have ever tried to analyze the traffic jam for macroscopic model considering the flow rate difference which have a significant impact on traffic movement. Based on this, a new lattice hydrodynamic model with the delayed feedback control will be presented to investigate its influence on traffic flow.

The outline of this paper is organized as follows. In Section 2, the lattice hydrodynamic model considering the flow rate difference is presented, and its stability condition is analyzed. In Section 3, a control signal will be added into the model proposed in Section 2 and delayed feedback control theory is used to analyze the stability conditions. In Section 4, several numerical simulations are carried out to verify the theoretical results. Conclusions are given in Section 5.

2. Lattice hydrodynamic model considering flow rate difference

To study the complex mechanism behind the traffic flow, Nagatani [27] put forward the lattice hydrodynamic model which meant the traffic flow could be represented with the product of the optimal velocity and average density as follows:

$$\begin{cases} \partial_t \rho + \partial_x(\rho v) = 0 \\ \partial_t(\rho v) = a\rho_0 v(\rho(x+\delta)) - a\rho v \end{cases} \tag{1}$$

where ρ and $\rho(x+\delta)$ are the local density at the position of x and $x+\delta$ at time t severally. ρ_0 means the local average density and δ is the average space headway which shows ρ_0 in an inverse way($\rho_0 = \frac{1}{\delta}$). a is the sensitivity of the driver and always greater than 0. v is the local average speed and $V(\rho)$ represents the optimal speed of traffic flow at density of ρ.

In order to make it easier to proceed the further study, Nagatani modified the equation in a discretization way as follows:

$$\begin{cases} \partial_t \rho_j + \rho_0 \left(\rho_j v_j - \rho_{j-1} v_{j-1}\right) = 0 \\ \partial_t(\rho_j v_j) = a\rho_0 v(\rho_{j+1}) - a\rho_j v_j \end{cases} \tag{2}$$

where the road is divided into N lattice sites and j indicates the site of the road on the one-dimensional lattice, ρ_j and v_j severally indicates the local density and the local average speed on site j at time t.

Recently, Redhu [42] proposed a delayed-feedback control for Nagatani's model in which flow evolution equation was modified taking the difference between current state and delayed one of site—$j+1$. The feedback gain was designed to suppress the traffic jam in lattice hydrodynamic model which finally verified the feasibility of feedback signal in suppressing the traffic congestion. Analogously flow rate difference is used to analyze the stability condition of the lattice hydrodynamic model. It is the dramatic change of flux and the formula is rewritten as:

$$\begin{cases} \partial_t \rho_{j+1} + \rho_0 \left(q_{j+1} - q_j\right) = 0 \\ \partial_t(q_j) = a\rho_0 v(\rho_{j+1}) - aq_j + u_j \end{cases} \tag{3}$$

where q represents the product of ρ and v. The flow rate difference u_j is described as:

$$u_j(t) = -\lambda M_j(t) = -\lambda \left[\frac{dq_j(t)}{dt} - \frac{dq_j(t-1)}{dt}\right] \tag{4}$$

with λ being flow rate parameter, the control signal describes the change of the flow rate between time t and $t-1$ at lattice j. And because Eq. (2) shows the similarity with Bando's car-following model, we adopt the analogous optimal speed equation:

$$V(\rho) = (v_{\max}/2)\left[\tanh(1/\rho - 1/\rho_c) + \tanh(1/\rho_c)\right] \tag{5}$$

where $v_{\max}$ and ρ_c severally denote the maximal speed and the critical safety density.

Later we suppose the desired density of vehicles and comprehensive flow are ρ^* and q^*, so the steady state of the following vehicles is:

$$[\rho_n(t), q_n(t)]^T = [\rho_n^*, q_n^*]^T. \tag{6}$$