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Propagation of structural uncertainty to linear aeroelastic stability

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ABSTRACT

The problem of linear flutter analysis in the presence of structural uncertainty is addressed. Whereas the propagation of uncertain structural parameters in finite element models has been carried out by a number of different methods, there appears to be less published work on the influence of random structural parameters on flutter speed. In this paper, we first evaluate the sensitivity of aeroelastic damping to a number of uncertain structural, geometrical and structural-damping parameters. The most significant parameters are identified and then randomised. Secondly, interval, fuzzy and probabilistic methods are used to propagate the structural uncertainty through the aeroelastic analysis resulting in regions of flutter-boundary uncertainty characterised by intervals, fuzzy membership functions and probability density functions. Interval analysis requires two optimisation procedures in order to find the bounds of the aeroelastic responses. The *Response Surface Method* (RSM) permits efficient optimisation and is used for the estimation of the gradient and Hessian. The resulting intervals are checked using *Monte–Carlo Simulation* (MCS). Probabilistic analysis is carried out using both first- and second-order perturbation, using the gradient and the Hessian determined by RSM. The first-order perturbation method is generally found to produce results in good agreement with the MCS, although there are differences at the tails of the distributions, especially for the unstable modes close to the flutter speed. The second-order perturbation method provides an improved prediction of the nonlinear behaviour at the tails. The flutter membership function predicted by the fuzzy method generally includes the nonlinear behaviour at the tails of the MCS distribution. Variability in structural mass and stiffness parameters is shown to have a significant effect upon the flutter intervals. Structural damping results in a small but significant increase in the flutter speed, but structural-damping variability does not translate into significant intervals of flutter-boundary uncertainty. Studies are carried out on the Goland wing, with and without structural damping, and on a generic fighter model.

1. Introduction

The accurate estimation of flutter boundaries is an important problem in aircraft certification. When the structural model includes parameter uncertainties, represented by intervals, fuzzy membership functions or probability density functions, then this uncertainty may be propagated through the aeroelastic model resulting in uncertain flutter boundaries, described correspondingly in terms of intervals, fuzzy memberships and probability densities. The review paper by Pettit [1] and references therein show the considerable attention that has already been paid to this subject. This paper is specifically concerned with aeroelastic analysis in the presence of structural uncertainty, and the evaluation of various propagation methods.

There are generally two classes of uncertainty, epistemic and aleatoric (irreducible) uncertainty [2]. The main cause of epistemic uncertainty is lack of knowledge, reducible by further information. Lack of confidence arising from either the choice of computational aeroelastic method or the fidelity of modelling assumptions is a form of epistemic uncertainty. Variability in structural parameters arising from the accumulation of manufacturing tolerances or environmental erosion is aleatoric. Structural variability must be characterised and the first step in achieving this is to discover which of the uncertain structural parameters have a significant affect on the aeroelastic analysis. The distribution or range of these parameters must be estimated. This variability may then be propagated through the model to determine a distribution or range of flutter speeds. In a small number of research papers [1] flutter-speed estimates are determined in the presence of parameter uncertainty. Poirion [3] used a first-order perturbation method to calculate the probability of flutter for given uncertainty in structural properties. The estimated flutter probability density function obtained by the perturbation method was found not to be in good agreement with MCS results. Kurdi et al. [4] used MCS to propagate the variation in dimensional properties of the structural parameters of the Goland wing in order to quantify the flutter-speed probability density function. Results showed the flutter speed to be

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Table 1
Nominal values of thicknesses and areas for the Goland wing finite element model.

Parameter	Thickness ft (m)	Parameter	Area ft^2 (m^2)
Upper and lower wing skins	0.0155 (0.0047)	Leading and trailing edge spar caps	0.0416 (0.003865)
Leading and trailing edge spars	0.0006 (0.00018)	Centre spar cap	0.1496 (0.013898)
Centre spar	0.0889 (0.0271)	Rib caps	0.0422 (0.003921)
Ribs	0.0347 (0.01058)	Posts	0.0008 (0.000074)

Table 2
Flutter speed bounds from different methods.

Mach	Lower bound of flutter speed ft/s (×0.3 048 m/s)					Mean flutter Speed ft/s (×0.3048 m/s)					Upper bound of flutter speed ft/s (×0.3048 m/s)				
	MCS	Pb 1st	Pb 2nd *n*	Pb 2nd *p*	Fuzzy	MCS	Pb 1st	Pb 2nd *n*	Pb 2nd *p*	Fuzzy	MCS	Pb 1st	Pb 2nd *n*	Pb 2nd *p*	Fuzzy
0.7	387.0	393.5	392.8	390.9	374.0	417.1	417.1	416.5	416.5	417.1	443.4	440.8	440.2	440.6	463.0
0.8	365.5	366.0	366.3	366.5	349.3	388.7	387.4	387.8	387.8	387.4	415.2	408.9	409.2	411.9	430.9
0.825	357.8	357.7	356.6	354.1	340.1	379.2	379.0	378.0	378.0	379.0	401.6	400.2	399.3	400.2	419.8
0.85	346.3	347.1	347.4	346.2	331.1	368.2	366.9	367.2	367.2	366.9	390.7	386.7	387.0	388.0	407.4
0.88	334.7	333.5	333.7	332.3	319.3	353.8	352.7	353.0	353.0	352.7	375.0	372.0	372.3	373.4	390.6
0.90	321.3	326.0	325.4	323.9	312.1	343.6	343.4	342.9	342.9	343.4	363.5	360.9	360.3	360.7	378.6
0.92	318.2	317.9	317.5	316.2	306.1	335.1	334.6	334.3	334.3	334.6	355.4	351.4	351.1	351.6	366.9
0.94	314.8	314.4	314.4	314.2	304.1	330.1	329.1	329.1	329.1	329.1	346.2	343.8	343.9	345.6	358.0
0.95	315.5	314.8	314.8	314.2	306.0	329.7	328.7	328.7	328.7	328.7	344.8	342.6	342.7	343.8	355.5
0.96	316.2	316.0	315.9	315.9	307.7	330.5	329.6	329.6	329.6	329.6	344.6	343.1	343.2	343.6	354.9

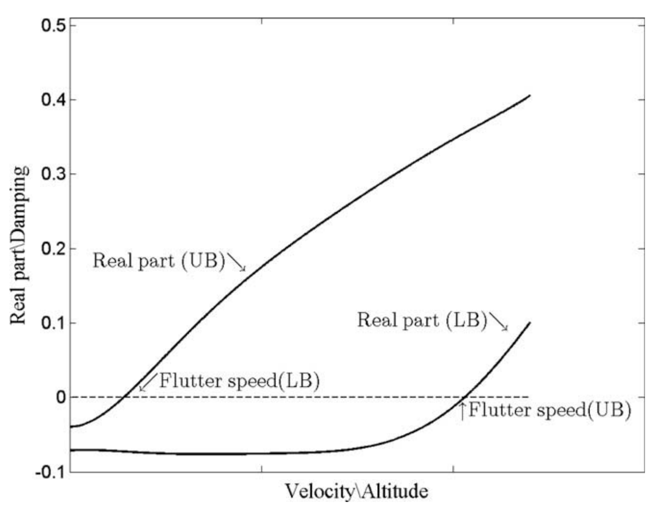

Fig. 1. Flutter speeds bounds and real parts of the flutter mode bounds.

highly sensitive to small changes in the structure. Attar and Dowell [5] used a response surface method to identify the effect of uncertainty on the response of a nonlinear aeroelastic system. Results were found to be in good agreement with those obtained by MCS. Wang et al. [6] considered the problem of flutter analysis in the presence of structural uncertainty using a CFD-based aerodynamic reduced-order model. They evaluated probability density functions for the flutter speeds of the Goland wing by randomizing the stiffness matrix. More recently, Verhoosel et al. [7] used stochastic finite element models to perform uncertainty and reliability analysis on fluid-structure stability boundaries. They found the sensitivity-based methods capable of characterising the statistical moments of the aeroelastic response.

In this paper a sensitivity study is carried out to select those uncertain structural parameters that influence the aeroelastic response considerably. Then three different approaches are considered for the characterisation of flutter-speed uncertainty. In the first approach, an interval flutter analysis is used. This method is said to be ‘possibilistic’ since no assumption is made about the probability distribution of either the structural parameters or the flutter speeds. Consequently the interval flutter method is restricted to the evaluation of upper and lower bounds without providing any information on how the uncertainty is distributed within such bounds. The interval flutter analysis requires a minimisation and a maximization of the aeroelastic response. The second approach makes use of fuzzy logic so that the uncertainty is defined according to a membership function. The fuzzy finite element method, introduced by Chen and Rao [8], has been used recently by Moens and Vandepitte [9] for the calculation of uncertain frequency response functions of damped structures. The fuzzy method is implemented within a number of α-levels for the numerical solution of the underlying interval finite element problem. Efficient optimisation procedures make use of the *Response Surface Method* (RSM) [10], which generally produces more accurate estimates of the gradient and Hessian than numerical estimation by finite differences. The third procedure is a probabilistic perturbation approach that makes use of the theory of quadratic forms [11,12]. Each solution of the flutter equation is perturbed about the mean values of the uncertain parameters through a truncated Taylor series expansion. Then the statistical moments of the aeroelastic responses are calculated. The procedure requires the calculation of the gradient and Hessian, which is estimated using RSM. When the perturbation is limited to the first-order terms of the Taylor series there is no need to calculate the Hessian matrix.

In the present article the three propagation methods are applied to the Goland wing [4] and to a model of a fighter aircraft. It is found in these examples that variability in structural damping has less effect on flutter speed intervals than does variability in structural mass and stiffness. Results achieved by first-order perturbation are found to be in good agreement with those obtained from MCS for the eigenvalues of those modes that do not contribute to the flutter. However there are differences at the tails of the distributions for the flutter modes, close to the flutter speed. The nonlinearity at the tails of the probability density functions can be estimated by both second-order perturbation and fuzzy methods. The study in reference [4] used MCS for propagation of structural uncertainty. This method is computationally expensive and may not be feasible for aeroelastic analysis using CFD. In this paper it is shown that the combination of interval analysis and RSM can be considered as a reliable and efficient tool for propagation of