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The impact of randomness on the distribution of wealth: Some economic aspects of the Wright–Fisher diffusion process

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HIGHLIGHTS

- Wealth concentration in random exchange models without redistribution.
- Capital taxation to prevent oligarchy.
- Economic interpretation of the Wright–Fisher diffusion process.

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ABSTRACT

In this paper we consider some elementary and fair zero-sum games of chance in order to study the impact of random effects on the wealth distribution of N interacting players. Even if an exhaustive analytical study of such games between many players may be tricky, numerical experiments highlight interesting asymptotic properties. In particular, we emphasize that randomness plays a key role in concentrating wealth in the extreme, in the hands of a single player. From a mathematical perspective, we interestingly adopt some diffusion limits for small and high-frequency transactions which are otherwise extensively used in population genetics. Finally, the impact of small tax rates on the preceding dynamics is discussed for several regulation mechanisms. We show that taxation of income is not sufficient to overcome this extreme concentration process in contrast to the uniform taxation of capital which stabilizes the economy and prevents agents from being ruined.

1. Introduction

1.1. Motivations

Since the seminal work of the Italian economist and sociologist Vilfredo Pareto [1], researchers have paid a lot of attention to describe, analyze and model wealth accumulation processes. In 1953, Champernowne [2] was the first to

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propose an exogenous multiplicative Markov chain model of stochastic wealth returns which generates Pareto-shaped wealth distributions. This paved the way for the so-called proportional random growth approach (see for example, [3,4] or [5]). More recently, equilibrium models were developed to study the characteristics of the wealth accumulation process as the result of agents' optimal consumption–saving decisions (see for example, [6] or [7]).

In this paper, we consider an economy simplified to the extreme and reduced to random games between agents. The games played are assumed to be fair in expectation. This situation may be seen as a basic model of randomness in the physical world or for modeling the effect of volatility on prices, considering that at least for the first order of magnitude the games may be assumed to occur with zero expectations. The focus is therefore on social and collective phenomena appearing in this purely speculative framework with or without regulation mechanisms. In some sense, our approach may be related to multiplicative Random Asset Exchange Models. These have recently been introduced in Econophysics (see [8] or [9], Chap. 8) to describe the evolution of wealth distribution in a population interacting economically. In fact, in these models, the interactions between two agents simply result in a well-chosen random redistribution of their assets, without taking into account the possible underlying micro-foundations. We follow this line to emphasize that, in our simple model, social inequalities are driven primarily by chance, rather than by differential abilities.

Our main objective is to study the dynamics of some elementary Markov games of chance in order to highlight the impact of random effects on the wealth distribution of interacting players. Surprisingly, even if zero-sum games of chance (expected to be fair) are played at any round, wealth distribution converges toward the maximum inequality case. This qualitative behavior has already been empirically observed in the literature (see [10] Chap. 15, [11,12] and [13]). But few papers have provided a theoretical framework to prove that luck alone may generate extreme disparities in wealth dynamics. Notable exceptions are [14] where the long time solution of a mean-field stochastic evolution equation for wealth is considered, [15] and [16] for the study of the so-called Yard sale model in Econophysics and [4,7] and [5] where investors are faced with an uninsurable idiosyncratic investment risk. In this paper, similar conclusions on wealth condensation are both supported by numerical and mathematical arguments, at the very least for small and high-frequency transactions in which Wright-Fisher diffusion processes naturally appear as limit models. To our knowledge, this is the first time this so-called family of stochastic processes, widely studied in theoretical population genetics (see [17] or [18]), is used to examine wealth concentration problems, providing interesting, new economic interpretations of these classical dynamics. We also investigate the impact of some regulation mechanisms on the qualitative behavior of the models considered. In particular, our findings provide a very simple agent-based framework for understanding how the Beta distribution, widely used in the literature as descriptive models for the size distribution of income [19,20], arises in wealth distribution problems.

Let us start by giving some elementary definitions and examples.

1.2. Fair elementary zero-sum games of chance (FEG)

We consider two players playing during $n \in \mathbb{N}^*$ consecutive rounds a zero-sum game of chance. If we denote by $(P_k^i)_{k\in\{1,\ldots,n\}}$ the payoff process (defined on the probability space $(\Omega, \mathcal{A}, \mathbb{P})$) of the player $i \in \{1, 2\}$, starting from a constant initial wealth X_0^i, we have $P_k^1 = -P_k^2$ (zero-sum game) and the wealth process is given by

$$X_k^i = X_0^i + \sum_{1 \leqslant j \leqslant k} P_j^i.$$

By a game of chance, we mean a game whose outcome depends on some random experiments. Contrary to what happens in game theory, we do not consider strategic interactions between players. The evolution of wealth only depends on a random redistribution mechanism between the interacting players through the payoff process. At this stage, our approach may be related to the Random Asset Exchange Models recently introduced in Econophysics (see [8] or [9], Chap. 8), to describe the evolution of wealth distribution in an economically interacting population. In fact, the economy is considered here in its simplest form: every time two players interact, wealth is transferred from one to the other, according to some elementary fixed rules, without taking into considerations their neoclassical foundations. Moreover, the zero-sum hypothesis implies that no wealth is imported, exported, generated, or consumed: wealth can only change hands. Working in a closed economic system will allow us to think in terms of proportions of wealth, instead of in absolute values.

Definition 1. We say that the preceding zero-sum game of chance is fair in expectation on $(\Omega, \mathcal{A}, \mathbb{P})$ (and we will write **FEG** for fair elementary game) if $\forall k \in \{1, \ldots, n\}$, $\mathbb{E}_{\mathbb{P}}[P_k^i] = 0$.

Remark 1. Let us emphasize that considering games fair in expectation is philosophically the most natural thing to do, if we have no additional reason for the presence of biases, and that it was indeed the first historical approach used in creating a prize-winning purse, adopted by Louis Bachelier in [21] who assumed that "the expectation of the speculator is zero".

In the next two examples, we suppose that $X_0^1 + X_0^2 = 1$ (reasoning in proportion of the total initial wealth $X_0^1 + X_0^2$ instead of using absolute values) and that each player bets a fixed amount a of his/her initial wealth.[2]

[2] For real world transactions, it is reasonable to consider the multiplicative exchange case (instead of the additive one) where the amount of money bet by each economic agent is proportional to their wealth, because wealthy people tend to invest more than the less wealthy. The parameter $1 - a$ may be seen as a measure of the propensity to save, assumed to be constant among all the participants, in order to ensure an identical involvement. For example, this kind of mechanism is considered in [13,14] or [8] in the framework of Econophysics.