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Improving interval analysis in finite element calculations by means of affine arithmetic

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ABSTRACT

Interval-based uncertainty models have proven to be well-suited for structural safety engineering with few data at hand. The practical use of interval analysis is hindered by the dependency problem, which leads to an overestimation of the uncertainty on the results. Affine arithmetic is a generalization of interval arithmetic that accounts for the relation between variables. By circumventing the dependency problem, it yields more accurate results. This paper presents a novel method to solve affine systems of linear equations, which allows for the application of affine arithmetic in finite element analysis. The proposed procedure is illustrated with three applications.

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1. Introduction

The continuous quest for better designs has stimulated the development and application of numerical models such as the finite element method. In order to specify all parameters of a detailed model, a large amount of information is required. Unfortunately, this information is not always available from the very beginning of the design process. In order to guarantee that the design fulfills its requirements under all circumstances, this uncertainty must be accounted for.

Two types of uncertainty are generally distinguished [13]: aleatory uncertainty and epistemic uncertainty. Aleatory uncertainty or inherent randomness concerns the uncertainty due to a natural variability. For instance, different samples from a mass production facility will not be exactly identical due to manufacturing tolerances, variable environmental conditions, etc. Epistemic uncertainty, on the other hand, originates from incomplete information or insufficient understanding of a phenomenon. For example, a bolted connection may be modeled with a rotational spring with an uncertain, but not random, stiffness.

Probability theory provides a sound mathematical framework to model aleatory uncertainty. An overview of stochastic methods in the context of computational mechanics is given by Schuëller [23]. Epistemic uncertainty can be treated with probability theory as well [3], but many researchers prefer to use different models for both kinds of uncertainty. As such, non-probabilistic uncertainty models have been developed, of which interval analysis and fuzzy set theory are the most common.

Interval analysis [16] was originally developed to model the propagation of roundoff errors through computerized calculations. Within the context of uncertainty modeling in structural design, it is used as a tool in anti-optimization [6,20,10,12]. This term expresses that the structure is designed under the assumption that the uncertain variables attain their most unfavorable values. This worst case scenario-based philosophy is quite appropriate for safety engineering, and implicitly present in many standards and codes.

Fuzzy set theory was introduced in 1965 by Zadeh [24], and has attracted the interest from scientists in a broad range of research fields. Initially intended to represent linguistic vagueness, its scope has been widened to epistemic uncertainty in general, providing a more intuitive alternative to probability theory. One of the reasons for its popularity is the wide range of interpretations attributed. Fuzzy set theory is closely related to possibility theory [25,5], it can be regarded as a multilevel interval analysis [1], and it can be used to represent expert knowledge [15]. Since the mathematical core of fuzzy calculations consists of interval analysis, both uncertainty models have known a largely parallel development.

This paper shows how interval analysis can be used in the static analysis of structures by means of the finite element method. In this context, an important drawback of interval variables is that throughout calculations, the dependency between different variables cannot be tracked and accounted for. As a consequence, the results of an interval analysis tend to be overly conservative. This is the so-called dependency problem which is discussed in detail in Section 2. In order to circumvent the dependency problem, af-

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fine arithmetic has been proposed as an enhancement of interval arithmetic [2]. Affine arithmetic offers a possibility to keep track of the dependency between variables throughout calculations. Section 3 presents a novel algorithm to solve an affine system of linear equations. Next, the advantages of affine arithmetic are illustrated with three case studies involving a finite element-based structural analysis.

2. Interval analysis

2.1. Definitions

In interval analysis, an uncertain variable is represented by a closed, finite interval. An interval variable $[x]$ is fully characterized by its lower bound $[x]^-$ and its upper bound $[x]^+$:

$$[x] = \{x \,|\, [x]^- \leqslant x \leqslant [x]^+\} \tag{1}$$

The information content of an interval variable is quite low: it represents a range of possible parameter values, without differentiating between these values. Therefore, interval variables are quite attractive when the amount of available information is limited. When more data are available, other uncertainty theories like probability theory or fuzzy set theory, that allow to take more information into account, become more appropriate.

The generalization of interval variables to interval vectors and interval matrices is straightforward. For instance, an interval vector $[\mathbf{x}]$ with independent entries $[x_1], \ldots, [x_n]$ is defined as:

$$[\mathbf{x}] = \begin{bmatrix} [x_1] \\ \vdots \\ [x_n] \end{bmatrix} \tag{2}$$

When a continuous mathematical function $f(x)$ is applied to an interval variable $[x]$, the result is the interval $[y]$ that includes the output $f(x)$ for every x in the interval $[x]$:

$$[y] = f([x]) = \{y = f(x) \,|\, x \in [x]\} \tag{3}$$

2.2. Interval arithmetic

The most common way to perform calculations on interval variables is by interval arithmetic. The core of interval arithmetic consists of a generalization of scalar arithmetic operators to interval arithmetic operators. For instance, the basic operators addition $(+)$, subtraction $(-)$, multiplication $(\times)$ and division $(/)$ are generalized for the case of interval variables as:

$$[x] + [y] = \left[[x]^- + [y]^-,\ [x]^+ + [y]^+\right] \tag{4a}$$

$$[x] - [y] = \left[[x]^- - [y]^+,\ [x]^+ - [y]^-\right] \tag{4b}$$

$$\begin{aligned}[x] \times [y] = \big[&\min\{[x]^-[y]^-, [x]^-[y]^+,\ [x]^+[y]^-, [x]^+[y]^+\}, \\ &\max\{[x]^-[y]^-, [x]^-[y]^+, [x]^+[y]^-,\ [x]^+[y]^+\}\big]\end{aligned} \tag{4c}$$

$$[x]/[y] = [x] \times \left[\frac{1}{[y]^+}, \frac{1}{[y]^-}\right] \quad \text{if } 0 \notin [y] \tag{4d}$$

Although some algebraic properties of operations on scalars hold equally for interval operations (for instance, commutativity and associativity of addition and multiplication), other properties only exist in a relaxed form. For example, the property of distributivity relaxes to a property which is called subdistributivity:

$$([x] + [y]) \times [z] \subseteq [x] \times [z] + [y] \times [z] \tag{5}$$

Furthermore, even when $0 \in [x] - [x]$ and $1 \in [x]/[x]$, a rigorous additive ar multiplicative inverse does not exist since $[x] - [x] \neq 0$ and $[x]/[x] \neq 1$.

The nonexistence of additive and multiplicative inverses and the relaxation of distributivity to subdistributivity are due to the dependency problem, which arises because different occurrences of a single interval variable in an expression are treated as independent variables. Consider for example a function $f(x) = \frac{1+x}{x}$ and an interval variable $[x] = [1, 2]$. The evaluation of $f([x])$ according to Eq. (4) leads to:

$$[y_1] = f([x]) = \frac{1 + [1, 2]}{[1, 2]} = \frac{[2, 3]}{[1, 2]} = [1, 3]$$

When the function $f(x)$ is simplified to $1 + 1/x$ prior to the evaluation of $f([x])$, however, the exact result is found:

$$[y_2] = f([x]) = 1 + \frac{1}{[1, 2]} = 1 + \left[\frac{1}{2}, 1\right] = \left[\frac{3}{2}, 2\right]$$

In the first calculation, the interval variable $[x]$ in the numerator and the denominator is treated independently. More specifically, the lower bound $[y_1]^- = 1$ is obtained when the value of x in the numerator is equal to the lower bound of the interval $[x]^-$ and, at the same time, the value of x in the denominator is equal to the upper bound of the interval $[x]^+$. The fact that the same variable appears twice is ignored, and results in an overestimation of the interval width of the result. This is avoided by the second evaluation, which is exact because $[x]$ is encountered only one time.

A mechanical analysis of a structure with the finite element method generally involves large systems of linear equations. Solving such systems with traditional methods such as Gauss-elimination or iterative methods requires numerous operations. Since the overestimation of the uncertainty due to the dependency problem strongly increases with the number of operations [2], these strategies are not appropriate for the solution of a linear system of interval equations.

Moreover, the dependency problem is encountered not only during the solution phase, but also in the assembly of the system matrices, since different elements may be influenced by the same interval variable [14]. As a consequence, it is not sufficient to develop an efficient algorithm that delivers sharp bounds on the solution of an interval system of linear equations, such as Hansen's method [8], Rump's inclusion algorithm [4,22], or the work by Neumaier [19]. Several authors [7,9,17,26,21,18] therefore propose strategies to incorporate the dependency between different element stiffness matrices. One of these methods is the element-by-element method of Muhanna et al. [17,18], which has been developed for the particular case where the element stiffness matrices depend linearly on an interval elasticity modulus. The global stiffness matrix is assembled by considering the different element stiffness matrices separately – thus circumventing the dependency problem – and imposes the constraints between elements with a Lagrange-multiplier based strategy.

2.3. Affine arithmetic

Affine arithmetic was introduced by Comba et al. [2] within the area of computational geometry as an enhancement of interval arithmetic. Manson [11] has applied this method to uncertainty modeling in structural analysis.

The key idea of affine arithmetic is to keep track of and account for the dependency between interval variables throughout the calculations. Consider a problem with n uncertain input variables $[\epsilon_1], \ldots, [\epsilon_n]$. Without loss of generality, assume that $[\epsilon_i] = [-1, 1]$, for $i = 1, \ldots, n$. Affine arithmetic then represents all variables during the analysis as:

$$\langle x \rangle = x_0 + x_1[\epsilon_1] + \cdots + x_n[\epsilon_n] + x_e[\epsilon_e] \tag{6}$$

in which x_0 is called the constant term of $\langle x \rangle$, which corresponds to the center value of the interval. The terms $x_i[\epsilon_i]$ express the linear

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