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Pinning control of clustered complex networks with different size

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ABSTRACT

In pinning control of complex networks, it is found that, with the same pinning effort, the network can be better controlled by pinning the large-degree nodes. But in the clustered complex networks, this preferential pinning (PP) strategy is losing its effectiveness. In this paper, we demonstrate that in the clustered complex networks, especially when the clusters have different size, the random pinning (RP) strategy performs much better than the PP strategy. Then, we propose a new pinning strategy based on cluster degree. It is revealed that the new cluster pinning strategy behaves better than RP strategy when there are only a smaller number of pinning nodes. The mechanism is studied by using eigenvalue and eigenvector analysis, and the simulations of coupled chaotic oscillators are given to verify the theoretical results. These findings could be beneficial for the design of control schemes in some practical systems.

1. Introduction

Many complex system can be represented by networks, and controlling the complex network is one of the most challenging problems in science and engineering fields. Moreover, the need of regulating the behavior of complex system with interacting units is a common feature of many natural and artificial systems. Over the past decade, great effort has been devoted to understanding the relationship between the network topology and dynamic process, and also designing the control strategies on them [1–3]. However, due to the complexity of network structure and the chaotic feature of node dynamics, the systems always become quite sensitive, making it difficult to be controlled. Hitherto, many control methods have been proposed to control chaotic systems [4–9], one of which is the pinning control that is now widely adopted in controlling chaotic networks [3,6,7,10–20]. For a network with a given coupling scheme, pinning control means to control the network state to a goal state or a specific targeting orbit by pinning only part of the system variables [6,7].

In order to tame a complex network through pinning control method, the control strategy should be carefully selected or specially designed. Usually, it is assumed that pinning the large-degree nodes can get a better control result [21–24], but this traditional pinning strategy may lose its efficiency when the network has a specific topology, e.g., modular structure. Many realistic networks have such structure, e.g., the network of books on American politics [25,26], Zachary's karate club network [27], and dolphin social network [28], here we call them clustered networks for simplicity. Traditional pinning control strategy may fail to deal with these networks, in certain situations, even random pinning strategy behaves better than it. Recently, Miao et al. [13] have studied the pinning controllability of clustered networks through Master Stability Function (MSF) and found that when the number of pinning nodes increases gradually, the ascending pinning scheme behaves more

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effectively than descending pinning scheme, which is similar to the results in the networks without modular structure [2]. However, in their study, all the clusters have similar size, while in many real networks, the size of cluster may be varied. For instance, the network about books on American politics can be roughly divided into three clusters, i.e., two relatively large clusters represent liberal and conservative, respectively, and one small cluster represents centrist or unaligned [26]. Such heterogeneity on cluster size requires us to propose better pinning control strategy.

In the present work, motivated by the above discussion, we will investigate the pinning control in clustered network with the cluster size highly varied. For the pinning control of a network, there are two basic questions: *Which nodes should be controlled? And how many nodes should be chosen?* These two questions will also be carefully investigated in this paper. In particular, we will study different pinning strategies in clustered networks, and by the method of eigenvalue and eigenvector analysis, we will show how the pinning location influences the pinning controllability. Our main finding is that, in the clustered networks with the cluster size highly varied, the traditional preferential pinning (PP) strategy behaves even worse than the random pinning (RP) strategy. Thus, we propose a new pinning strategy, namely cluster pinning (CP) strategy, which generally behaves much better than the PP and RP strategies in this kind of networks. Interestingly, we find that the pinning location can be easily observed in the eigenvector space and these eigenvector components can help to inference the critical eigenvalue which represents the controllability of the system. These findings have the potential to extend our knowledge of pinning control in the clustered networks and can be of utility in diverse domains such as disease control [29] and air traffic control [30].

The rest of our paper are organized as follows: in Section 2 we give our model of clustered network pinning control and study the phenomena by applying control strategies in real and artificial network. In Section 3, we study the mechanism of pinning strategy by using the method of eigenvector analysis. In Section 4, we present our new pinning strategy and compare it with other pinning strategies. Finally in Section 5, we provide our discussion and conclusion.

2. Method and observation

We consider the following model with the network of pinning control:

$$\dot{\mathbf{x}}_i = \mathbf{F}(\mathbf{x}_i) + \varepsilon \sum_{j=1}^{N} a_{i,j}[\mathbf{H}(x_i) - \mathbf{H}(x_j)] + \eta \sum_{m \in V} \delta_{im}[\mathbf{H}(\mathbf{x}_T) - \mathbf{H}(\mathbf{x}_i)] \tag{1}$$

where $i, j = 1, 2, \ldots, N$ are node indices, $\mathbf{F}$ is the function of node dynamics and $\mathbf{H}$ is the coupling function, ε and η represent the uniform coupling strength and the pinning strength, respectively. The network structure is captured by the adjacency matrix $\mathbf{A}$, with $a_{i,j} = 1$ if nodes i and j are directly connected and $a_{i,j} = 0$ otherwise. Here, the network is supposed to be undirected and unweighted, therefore the degree of node i can be read as $k_i = \sum_{j=1}^{N} a_{i,j}$. $\mathbf{x}_T$ is the target orbit, that is, the assumed controlled state of the whole network. Denoting f as the ratio of the pinning nodes to all nodes of the network, thus the number of pinning nodes is $N_p = f \times N$, then the set of pinning nodes is denoted by $V = \{n_i\}$, with $i = 1, \ldots, N_p$ as the indices of these pinning nodes. Specifically, if node i is pinned in the network, we have $\delta_{im} = 1$ in Eq. (1), otherwise, $\delta_{im} = 0$. Here, for simplicity, the dynamic of controller is set the same as the network nodes, i.e. $\dot{\mathbf{x}}_T = \mathbf{F}(\mathbf{x}_T)$.

Treating the controller as an additional node of the network, the pinning problem can be considered as a network synchronization problem and the original system can be embedded into an extended network with $(N + 1)$ nodes [22]. Under such consideration, the enlarged network can be characterized by the following matrix:

$$B = \begin{pmatrix} a_{11} & a_{12} & \cdots & a_{1N} & b_{1(N+1)} \\ \vdots & \vdots & \ddots & \vdots & \vdots \\ a_{i1} & a_{i2} & \cdots & a_{iN} & b_{i(N+1)} \\ \vdots & \vdots & \ddots & \vdots & \vdots \\ a_{N1} & a_{N2} & \cdots & a_{NN} & b_{N(N+1)} \\ 0 & 0 & 0 & 0 & 0 \end{pmatrix}.$$

This enlarged network can be regarded as a special weighted and directed network. In this network, the controller has the index $N + 1$, with the value $b_{i(N+1)} = \eta/\varepsilon$ in the last column if $i \in V$ and $b_{i(N+1)} = 0$ otherwise. Generally, the linear stability of the target synchronization state $\{\mathbf{x}_i(t) = \mathbf{x}_T,\ i = 1, 2, \ldots, N\}$ can be analyzed by MSF [31–33], which shows the synchronizability of a network is generally determined by the extreme eigenvalues of the network coupling matrix. Then, Eq. (1) can be converted and diagonalized into an enlarged dynamic system with $N + 1$ decoupled blocks:

$$\dot{\mathbf{y}}_i = [\mathbf{DF}(\mathbf{x}_T) - \varepsilon\lambda_i \mathbf{DH}(\mathbf{x}_T)]\mathbf{y}_i, \tag{2}$$

where $\mathbf{y}_i,\ i = 1, 2, \ldots, N + 1$ represent the modes that are transverse to the synchronous manifold $\mathbf{x}_T$, and $\mathbf{DF}(\mathbf{x}_T)$ and $\mathbf{DH}(\mathbf{x}_T)$ are the Jacobian matrices of the corresponding vector functions evaluated on $\mathbf{x}_T$. $0 = \lambda_1 < \lambda_2 \leq \cdots \leq \lambda_{N+1}$ are the eigenvalues of the coupling matrix $\mathbf{C} = \mathbf{B} - \mathbf{DI}$, $\mathbf{D} = (d_1, d_2, \ldots, d_{N+1})^T$, where $d_i = \sum_{j=1}^{N+1} b_{ij}$, is the coupling intensity of node i and $\mathbf{I}$ is the identity matrix of dimension $N + 1$. The coupling matrix $\mathbf{C}$ is Laplacian matrix and according to Ref. [31], this matrix satisfies $\sum_{j=1}^{N+1} C_{ij} = 0$, which is referred to as a diffusive condition. The mode associated with