Physica A 478 (2017) 177–187

Contents lists available at ScienceDirect

Physica A

journal homepage: www.elsevier.com/locate/physa

Asymptotic properties of restricted naming games

Biplab Bhattacherjee [a], Amitava Datta [b], S.S. Manna [a,*]

[a] Satyendra Nath Bose National Centre for Basic Sciences, Block-JD, Sector-III, Salt Lake, Kolkata-700106, India
[b] School of Computer Science and Software Engineering, University of Western Australia, Perth, WA 6009, Australia

HIGHLIGHTS

- Naming games are studied with finite sizes of the agent vocabularies.
- Naming games are studied with limited number of distinct names.
- Different dynamical rules lead to different new power law exponents.

ARTICLE INFO

Article history:
Received 16 October 2016
Received in revised form 16 February 2017
Available online 28 February 2017

Keywords:
Naming games
Self-organized systems
Scaling
Critical exponents
Structures and organization in complex systems
Critical phenomena

ABSTRACT

Asymptotic properties of the symmetric and asymmetric naming games have been studied under some restrictions in a community of agents. In one version, the vocabulary sizes of the agents are restricted to finite capacities. In this case, compared to the original naming games, the dynamics takes much longer time for achieving the consensus. In the second version, the symmetric game starts with a limited number of distinct names distributed among the agents. Three different quantities are measured for a quantitative comparison, namely, the maximum value of the total number of names in the community, the time at which the community attains the maximal number of names, and the global convergence time. Using an extensive numerical study, the entire set of three power law exponents characterizing these quantities are estimated for both the versions which are observed to be distinctly different from their counter parts of the original naming games.

1. Introduction

The aim of the model of naming game is to study the evolution of consensus opinion in the context of naming a single object in a large community of agents [1,2]. Different agents refer to the object using different names when the object is introduced initially. Agents interact among themselves and share the names that have been already introduced according to a set of specific rules. At the early stage, the number of distinct names for the object increases as the agents introduce new names for the object. However as the game progresses, a consensus name gradually emerges and distinct names disappear. The dynamical evolution of the game terminates when all agents agree upon a single name through mutual interactions and following the rules of the game.

At an arbitrary intermediate stage, an agent has a number of names of the same object in his vocabulary suggested by different groups of agents. An agent, under the sharing dynamics, not only learns new names for the object but also shares names from his own vocabulary with other agents. In the models studied for the dynamics of naming games in the literature, the sizes of the vocabularies of the agents have been assumed to be infinite [3–10]. Real world data in this problem

* Corresponding author.
E-mail address: manna@bose.res.in (S.S. Manna).

http://dx.doi.org/10.1016/j.physa.2017.02.070

have been analyzed in [11]. It has also been shown that faster convergence can be attained in presence of overhearers [12]. Categorization of different names and color naming have been studied as well [13–15].

However, in reality, an individual agent has only a finite amount of memory. Therefore, it would be quite appropriate to study the effect of finiteness of the vocabulary sizes in the dynamics of naming games. In this work, the vocabulary size of every agent is assumed to be finite and has been restricted to a certain fixed cut-off value, which has been suitably tuned.

The dynamics of naming game is defined in terms of a sequence of bipartite interactions in a community of N agents. A 'pseudo' time t is defined for the convenience of following the dynamics, which is equal to the number of bipartite interactions. In each interaction, a pair of distinct agents is randomly selected and are allowed to interact. At any intermediate time, the vocabulary of each agent i is likely to have some entries and is denoted by the set $\{\ell_i(t)\}$ of size $\ell_i(t)$. Commonly, the naming dynamics is described in terms of a few quantities. For example, the total number of names $W(t, N) = \Sigma_{i=1}^{N} \ell_i(t)$ with all agents at time t is a well known quantity to look at. Typically, this number initially grows with time, reaches a maximum, which then gradually decreases and finally converges to N. The maximum value of W after the configuration averaging grows with N as a power law like,

$$\langle W_m(N)\rangle \sim N^{\gamma}. \tag{1}$$

At the same time, it is also customary to define two different time scales associated with the evolution dynamics. One is the time t_m when W reaches its maximum value. Again, a configuration averaged value of this quantity grows like

$$\langle t_m(N)\rangle \sim N^{\alpha}. \tag{2}$$

Secondly, one defines the convergence time t_f when every agent has only one name in his vocabulary, the same name for all agents and therefore the entire community has only N names. The averaged value of such a time scale is also assumed to vary like

$$\langle t_f(N)\rangle \sim N^{\beta}. \tag{3}$$

The three exponents in the power, namely α, β and γ, characterize the naming game. In the version of the game stated above, an agent can interact with any other agent. Therefore it is a mean field model. Yet, we have seen in the literature that the values of the above set of exponents do depend on few detailed features of the dynamical rules, as mentioned below.

In Section 2, we have studied both the symmetric and asymmetric naming games with restricted size of the agents' vocabulary and observe that the set of exponents cross over to a new set of values not studied earlier in the literature. Further in Section 3, we have studied the effect of limiting the initial number of names that are assigned to the agents. In Section 4 we summarize.

2. Symmetric and asymmetric naming games with restricted vocabulary

The naming game is defined in terms of a community of N agents and a new object to name [1]. An agent, either invents a new name for the object, or he learns a name from another agent by the bipartite sharing dynamics. Finally, all the agents come to an agreement and refer the object by a single name. This spontaneous evolution of a consensus name is the objective of the naming game. During the time evolution, pairs of randomly selected distinct agents execute a sequence of interactions to share their stock of the names of the object.

In the literature, two models of naming game have been studied. In the original 'asymmetric' naming game, one of the two agents selected for sharing, say the 'i'th agent, is called the 'speaker', where as the 'j'th agent is termed as the 'hearer' [1]. The speaker first randomly selects a name from his vocabulary and checks if the hearer also has the same name. If the hearer has this name it is called a successful sharing and then the vocabulary sizes of both the agents are reduced to unity, both having only the selected name. On the other hand, in case of a failure, the selected name of the speaker is added to the vocabulary of the hearer.

In comparison, in the 'symmetric' naming game [10], there is no distinction between the speaker and the hearer. Here, for a successful move, the entire subset of names that are common in the vocabularies of the agents i and j are retained, and the remaining un-common names are deleted from the vocabularies of both the agents. On the other hand, in case of a failure there is no common name and both the agents get the combined list of both agents' vocabularies. There are studies of similar symmetric exchange of informations in the literature [16–19].

We have studied the effect of restrictions on both the symmetric and asymmetric naming games. We describe the game and present the plots of the data for the symmetric naming game only. The plots of the asymmetric naming game exhibit very similar type of behavior. However, the power law exponents of both the games are found to be distinctly different and we have enlisted them in Table 1.

2.1. The model

In the symmetric game, we first abolish the step for the invention of names. Instead, we assign every agent a distinct name at the initial stage. Therefore, $\ell_i(t) = 1$ for all i at time $t = 0$ and the dynamics starts with N such distinct names. Further, we apply a restriction to the vocabulary size of every agent. The vocabulary size is assigned a maximal cut-off value s, same