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Complex motion of a vehicle through a series of signals controlled by power-law phase

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HIGHLIGHTS

- We studied the dynamic motion of a vehicle through the series of signals controlled by the power law phase.
- We described the vehicular motion in terms of the nonlinear map model.
- We explored that the vehicle exhibits the periodic and irregular motions by varying the power of the signal phase.

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ABSTRACT

We study the dynamic motion of a vehicle moving through the series of traffic signals controlled by the position-dependent phase of power law. All signals are controlled by both cycle time and position-dependent phase. The dynamic model of the vehicular motion is described in terms of the nonlinear map. The vehicular motion varies in a complex manner by varying cycle time for various values of the power of the position-dependent phase. The vehicle displays the periodic motion with a long cycle for the integer power of the phase, while the vehicular motion exhibits the very complex behavior for the non-integer power of the phase.

1. Introduction

Engineers have extensively investigated transportation problems so far. Recently, physicists have been interested in traffic problems [1–5]. They have applied the concepts and techniques of physics to such complex systems as transportation systems. Physics, other sciences and technologies meet at the frontier area of interdisciplinary research. The traffic flow and pedestrian flow have been studied from a point of view of statistical mechanics and nonlinear dynamics [6–45].

In city traffic, vehicles are controlled by traffic signals to give priority for a road because they encounter at crossings. The vehicular traffic depends highly on the control method of traffic signals. Generally, the signals are controlled by either synchronized or green-wave strategies [46–51]. In the synchronized strategy, all the signals change simultaneously and periodically where the phase has the same value for all signals. In the green-wave strategy, the signal changes with a certain time delay between the signal phases of two successive intersections where the phase difference between two signals has the same value for all signals. The vehicular traffic controlled by signals depends highly on the signal's strategy. The operator will be able to control the traffic signal by the use of various strategies. One can manage both cycle time and phase of signals to control the vehicular traffic. The traffic flow at the synchronized and green-wave strategies has been studied extensively. The vehicular traffic at the synchronized strategy is controlled only by the cycle time. While the vehicular traffic at the green-wave strategy is controlled by both cycle time and phase shift.

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The vehicular motion at both synchronized and green-wave strategies shows the periodic behavior. One can study the traffic flow at both synchronized and green-wave strategies by the traffic model taking into account a few signals because of the periodic motion [45,46]. Generally, one controls the traffic flow by adjusting the phase shift among signals. In the case, the phase of signals depends on the position of signals. By adjusting the phase among signals, one can control the stopping time at signals. In the result, the vehicular motion displays a complex behavior. The traffic flow controlled by signals with the position-dependent phase has been little investigated until now [47–51]. One cannot study the traffic flow through a series of signals controlled by the position-dependent phase using the traffic model with a few signals because the traffic depends on all signals. In order to study the dynamic behavior of the vehicle, it is necessary and important to take into account the infinite series of signals on an open boundary [47–51]. The power-law phase is considered as the position-dependent phase. It is not known how the traffic flow varies with the power of the position-dependent phase. How can the operator control the traffic flow by the power-law phase?

In this paper, we study the dynamic motion of a vehicle through an infinite series of signals controlled by the power-law phase. We present the nonlinear-map model for the vehicular motion through the series of signals with the position-dependent phase. We show how the vehicular motion is controlled by the power-law phase. We clarify the dynamical behavior of a single vehicle through the series of signals by varying both cycle time and power-law phase.

In Section 2, we propose the nonlinear-map model for the vehicular motion through an infinite series of signals controlled by the position-dependent phase. In Section 3, we present the numerical result for various power-law phases. We show that the vehicular motion displays a complex behavior. We present the summary in Section 4.

2. Nonlinear-map model

We consider the motion of vehicles going through the infinite series of traffic lights on the single-lane roadway at a low density. It is known that the vehicular traffic is in the free traffic state at a low density where all vehicles move almost with the maximal speed and a vehicular motion is little affected by other vehicles. Therefore, one can assume that vehicles are not correlated each other at a low density. Here, we consider the motion of a single vehicle going through the infinite series of traffic lights on the single-lane roadway.

The signals are numbered by $1, 2, 3, \ldots, n, n+1, \ldots$ to x direction. The signals are positioned with the same interval on the roadway where the interval between signals $n-1$ and n is indicated by l. All signals change periodically with period t_s. Period t_s is called as the cycle time. The phase of signals varies with signals. The vehicle moves with the mean speed v between a signal and its next signal. All signals change periodically from red (green) to green (red) with a fixed time period $(1-s_p)t_s$ (s_pt_s). The period of green is s_pt_s and the period of red is $(1-s_p)t_s$. Fraction s_p represents the split which indicates the ratio of green time to cycle time.

Each signal timing is controlled by offset time t_{offset}. The offset time means the difference of phases between two successive signals. In the green wave (delayed) strategy, the phase shift of signal n is given by $t_{phase}(n) = nt_{offset}$ where the phase at signal n is indicated by $t_{phase}(n)$. Then, the signal switches from red to green in green wave way. The phase increases with signal n. Generally, the signal phase depends on the signal position. Here, we study the position-dependent phase with power law. Then, the phase is given by

$$t_{phase}(n) = \alpha n^{\beta}, \tag{1}$$

where α is a constant and β is the power. When $\beta = 0$, the phase is the same for all signals and the case corresponds to the synchronized strategy. If $\beta = 1$, the phase difference between two successive signals is the same for all signals and the case corresponds to the green-wave strategy.

When a vehicle arrives at a signal and the signal is red, the vehicle stops at the position of the signal. Then, when the signal changes from red to green, the vehicle goes ahead. We define the arrival time of the vehicle at signal n as $t(n)$. The arrival time at signal $n+1$ is given by

$$t(n+1) = t(n) + l/v + \{r(n) - t(n)\}. \tag{2}$$

Here, l/v is the time it takes for the vehicle to move between signals n and $n+1$. $r(n)$ is such time that signal n just changed from red to green. Since signals vary with period t_s(cycle time), it is described by

$$r(n) = t_s\{\mathrm{int}\left((t(n) + t_{phase}(n))/t_s\right) + 1\} - t_{phase}(n). \tag{3}$$

Function $r(n)$ is the periodic function with cycle t_s and phase $t_{phase}(n)$. On the other hand, when a vehicle arrives at a signal and the signal is green, the vehicle does not stop at the signal and goes ahead without changing speed. The arrival time at signal $n+1$ is given by

$$t(n+1) = t(n) + l/v. \tag{4}$$

By unifying Eqs. (1)–(4), we have

$$\begin{aligned} &t(n+1) = t(n) + l/v + \{r(n) - t(n)\} \times H[t(n) + t_{phase}(n) - t_s\{\mathrm{int}((t(n) + t_{phase}(n))/t_s)\} - s_pt_s] \\ &\quad \text{with } r(n) = t_s\{\mathrm{int}\left((t(n) + t_{phase}(n))/t_s\right) + 1\} - t_{phase}(n) \\ &\quad \text{and } t_{phase}(n) = \alpha n^{\beta}, \end{aligned} \tag{5}$$