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Geometric Algebra and Information Geometry for Quantum Computational Software

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The art of quantum algorithm design is highly nontrivial. Grover's search algorithm constitutes a masterpiece of quantum computational software. In this article, we use methods of geometric algebra (GA) and information geometry (IG) to enhance the algebraic efficiency and the geometrical significance of the digital and analog representations of Grover's algorithm, respectively. Specifically, GA is used to describe the Grover iterate and the discretized iterative procedure that exploits quantum interference to amplify the probability amplitude of the target-state before measuring the query register. The transition from digital to analog descriptions occurs via Stone's theorem which relates the (unitary) Grover iterate to a suitable (Hermitian) Hamiltonian that controls Schrodinger's quantum mechanical evolution of a quantum state towards the target state. Once the discrete-to-continuos transition is completed, IG is used to interpret Grover's iterative procedure as a geodesic path on the manifold of the parametric density operators of pure quantum states constructed from the continuous approximation of the parametric quantum output state in Grover's algorithm. Finally, we discuss the dissipationless nature of quantum computing, recover the quadratic speedup relation, and identify the superfluity of the Walsh-Hadamard operation from an IG perspective with emphasis on statistical mechanical considerations.

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