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# Evaluation of high-order concentration statistics in a dispersing plume

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## HIGHLIGHTS

- A numerical stochastic model for turbulent dispersion is presented and tested against wind tunnel data.
- The comparison is performed for concentration statistics up to the fourth order.
- The results of the model allow analyzing the relationship among the moments up to the sixth order.
- The scaling of the higher order moments is verified.

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## ABSTRACT

In this paper we analyze the concentration higher-order moments of a dispersing plume in a neutral wind tunnel boundary layer. They are evaluated through a fluctuating plume model in which the plume barycenter is directly calculated from the measured mean concentration field. The reliability of the model is firstly verified by comparing the simulated and measured moments up to the fourth order. Then the relationships between higher and lower-order moments are investigated seeking for a proper scaling of the fourth, fifth and sixth-order normalized moments. We found that a general behavior appears when they are expressed as a function of the third-order moments. In this case each order calculated or measured at different distances collapses on the same curve.



## 1. Introduction

The study of the concentration probability density function (PDF) and of its moments is central in the field of Lagrangian dispersion modeling. Since a PDF may be evaluated through its moments, it is important to verify the relationship among the higher-order moments and their implication on the PDF shape. For practical purposes, the concentration PDF is often considered having a Gaussian distribution and thus being completely defined by its mean and variance. In Eulerian state-of-the-art models the choice of a Gaussian PDF corresponds to the simple down-gradient approximation for the evaluation of the turbulent fluxes [1]. Hence, turbulence is assumed to be local and turbulent fluxes at a given height are evaluated through the local gradient of the mean field at the same height, neglecting the contribution of the third order moments. This hypothesis is only justified for very simple flows when the turbulent mixing length is much less than the length scale of heterogeneity of the mean flow and it is not appropriate if applied to convective fluxes since it does not properly take into account more complex mixing processes [2–5]. Either in convective conditions or in a sheared boundary layer, turbulence

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is non-local and its structure in the vertical direction is far from Gaussian and this leads to a more complex concentration PDF. The non-local properties of the flow are related to the skewness of the velocity field and thus [6] suggested that, to adequately simulate the concentration PDF of a dispersing plume, the third-order moment of the concentration field is needed.

However, estimating high-order moments is still a challenging task and a literature review [e.g., 7] reveals that there is no agreement on their scaling properties. According to [6], higher-order moments collapse on a single curve when plotted against the skewness, but show no dependency on the normalized second-order moment. [8] showed that concentration values measured at isolated points only depends on the skewness. On the contrary, more recent atmospheric analysis performed on a large concentration fluctuations dataset [e.g. 7,9–11] showed that the higher-order moments of the concentration field depend on the second-order normalized concentration moment. These discrepancies are important since the different scaling of the high-order moments may have consequences in the parameterization of the concentration PDF in advanced dispersion models.

Despite its central role in olfactory research, in the chemistry of naturally emitted volatile organic compound and in toxic, flammable or explosive releases, the prediction of higher order moments of the concentration in Lagrangian stochastic modeling is still an open problem. Classical approaches to the concentration fluctuations and to the higher order moments are the two-particle models [e.g. 12,13], the meandering and fluctuating plume models [e.g. 14–18] and the Lagrangian PDF methods [e.g. 19,20,11]. Following [21] who introduce the concept of "concentration deficit", [22,23] developed single particle Lagrangian models able to simulate the concentration variance assigning a time dependent quantity to each particle. In a different way, [24] uses the diffusion–advection equation to create a fictitious field of concentration variance sources able to simulate the concentration fluctuations in real scenarios.

In this paper, starting from the work of [15] we develop an offline version of the fluctuating plume model (FPM) [25] able, in principle, to evaluate all the moments of the concentration PDF if the mean concentration field is known.

In the following we first verify the FPM against the Nironi et al. [26] data also looking for its limits and weaknesses. Then, we use the calculated higher order moments to assess some similarity relationships proposed in the literature.

In Sections 2 and 3 the model equations are presented. In Section 4 the FPM is validated against a new comprehensive concentration fluctuation dataset [26]. The consistency of the model formulation is further tested in Sections 5 and 6 where the relationships between the higher-moments are analyzed.

## 2. The fluctuating plume model offline approach

In the model the concentration fluctuations are calculated off-line respect to the mean concentration dispersion. In the FPM approach, the ensemble dispersion of a plume is viewed as the sum of a number of instantaneous plumes. In other words, the absolute dispersion is divided in two components: the meandering of the instantaneous plume and the relative diffusion around its instantaneous centroid. The motion of the centroid of each instantaneous plume is considered in a fixed coordinate system relative to the source, whereas the concentration distribution within the instantaneous plume is calculated relatively to the plume centroid. In this way, the effect of the large scale is taken into account by the centroid statistics that could account for the skewness. On the other side, the relative dispersion is locally resolved around the centroid and its statistics can be easily parameterized. Following [27], the concentration PDF, $p_c$, can be written as:

$$p_c(c; x, y, z) = \int_{-\infty}^{\infty}\int_{0}^{\infty} p_{cr}(c; x, y, z, y_m, z_m)p_m(x, y_m, z_m)dy_m dz_m \tag{1}$$

where $p_m(x, y_m, z_m)$ is the PDF of centroid position $(y_m, z_m)$, $p_{cr}(c; x, y, z, y_m, z_m)$ is the relative concentration PDF in a reference frame relative to $(y_m, z_m)$. If we assume statistical independence of the dispersion along the two horizontal directions, it is possible to factorize the centroid PDF as:

$$p_m(x, y_m, z_m) = p_{ym}(x, y_m)p_{zm}(x, z_m). \tag{2}$$

The concentration moment of order $n$, $\overline{c^n}(x, y, z) = \int_0^\infty c^n p_c(c; x, y, z)\, dc$, is given by:

$$\overline{c^n}(x, y, z) = \int_{-\infty}^{\infty}\left[\int_{0}^{\infty}\int_{0}^{\delta} c^n p_{cr}(c; x, y, z, y_m, z_m)\, dc\,\right] p_{ym}(x, y_m)\, p_{zm}(x, z_m)\, dy_m\, dz_m \tag{3}$$

where $\delta$ is the boundary layer height. Eq. (3) exemplifies the principle of the fluctuating plume model, where the concentration field is evaluated from the superposition of the meandering of the plume centroid, $p_m(x, y_m, z_m)$, and the relative concentration statistics:

$$\overline{c}_r^n(x, y, z, y_m, z_m) = \int_0^\infty c^n p_{cr}(c; x, y, z, y_m, z_m)\, dc$$

where $p_m(x, y_m, z_m)$ should be simulated and $\overline{c}_r^n(x, y, z, y_m, z_m)$ should be parameterized.

In order to calculate the mean concentration field, most of the FPM recent versions, are coupled with a Lagrangian Stochastic Model for the particle trajectories. On the contrary, in the present version we use the [15] approach. The