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Global oscillations in the Optional Public Goods Game under spatial diffusion

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HIGHLIGHTS

- We studied spatial diffusion in optional public goods game;
- We quantify the global oscillations (RPS cycles) by proposing a new amount;
- High mobility and low occupation enlarge the occurrence of global oscillations;

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ABSTRACT

Social dilemmas lead to natural conflict between cooperation and self interests among individuals in large populations. The emergence of cooperation and its maintenance is the key for the understanding of fundamental concepts about the evolution of species. In order to comprehend the mechanisms involved in this framework, here we study the Optional Public Good Games with focus on the effects of diffusive aspects in the emergent patterns of cyclic dominance between the strategies. Differently from other works, we showed that rock–paper–scissors (RPS) patterns occur by introducing a simple kind of random mobility in a lattice sparsely occupied. Such pattern has been revealed to be very important in the conservation of the species in ecological and social environments. The goal of this paper is to show that we do not need more elaborated schemes for construction of the neighbourhood in the game to observe RPS patterns as suggested in the literature. As an interesting additional result, in this contribution we also propose an alternative method to quantify the RPS density in a quantitative context of the game theory which becomes possible to perform a finite size scaling study. Such approach can be very interesting to be applied in other games generically.

1. Introduction

The evolution of cooperation, fairness, or pro-social behaviour among non-related individuals is one of the fundamental problems in biology and social sciences [1]. Reciprocal altruism fails in providing a good solution if iterations are not repeated. Punishment can be effective in few games as the iterated ultimatum game (UG), where a player (proposer) suggests a division of some food for example that can or cannot accepted by other player (responder) and offers from fifty–fifty division [2–6] are rejected. However this mechanism, in the case of prisoner dilemma (PD) or even public goods games (PGG) requires that defectors must be identified as observed in Ref. [7]. In such games, players can cooperate or defect to obtain their payoffs, and the defection, particularly, that can seem the best alternative for a player in one or two rounds, can deteriorate its payoff

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and of its "partners" at long terms. In PGG, particularly, all players can contribute or not with a defined quantity for the public good. The sum of contributions is duplicate or triplicate or more generally multiplied by a profit factor r and then all money and dividends are divided among all players. But the contribution is voluntary which leads to the free rider problem or the under-provision of the goods or services.

Optional participation in the PGG is a simple but effective mechanism that can avoid possible exploiters and overcome the social dilemma [8,9], since the cooperators and defectors can coexist due to the abstention alternative. These works as well as many others (see for example [10–12]) consider a dynamics with many public games, where each one of them corresponds to a different neighbourhood and its central node, differently from some alternative works (not so explored in literature) that consider the dynamics of an only single and large public goods game with interacting players (see for example [13,14]).

Therefore, the so called Optional Public Goods Game (OPGG) can provide an useful representation of many social conflicts in which the cooperation plays an important role in the good operation of general public services. Voluntary participation in PGG may provide a way to keep stable and persistent levels of cooperation, without secondary mechanisms as punishment or reward [15]. In the stationary state of OPGG dynamics, the coexistence of the three strategies: cooperators, defectors, and loners, as well as dominance cycles of each one of these strategies in sequence, i.e., the so called rock–paper–scissors (RPS) regime, were reported as solutions of the mean field replicator dynamics [16], as well as for simulation in different topologies [11,12,17].

Other important aspects may influence the cooperation patterns in PD and PGG and among them, one has called the attention of physicists that study evolutionary game theory: the mobility of the players [18]. Mobility is an interesting mechanism to evaluate if a social or biological system preserve its environment or biodiversity by considering the different strategies in the population [18,19], or by simply changing the critical rates in epidemiological systems simulated by cellular automata [20]. Thus, the investigation of RPS pattern which is an interesting case of emergence of cooperation, deserves more attention in OPGG and in this point we will be asking for: Is the mobility an important ingredient to influence or even preserve the RPS patterns in OPGG? If this is the case, for which occupation this can happen?

In this paper we propose to study the effects of mobility in OPGG. We focus our investigation in three different contributions:

1. First, we would like to answer about the connection between the mobility and RSP patterns observed in OPPG and in other game theory protocols;
2. In square lattices, where each site can be occupied by only one player, we intend to explain how the occupation (density of occupied sites) and mobility characterized by a simple diffusion parameter p, defined as probability that a player moves to a empty site, randomly chosen among nearest neighbours, influence the RSP patterns;
3. By following this investigation, we would like to propose a parameter to measure the density of RPS patterns in Game theory, more precisely in PD and PGG with voluntary participation, or even in other game theory protocols;

In this contribution we present an analysis, by means of Monte Carlo Simulations, looking for the coexistence of the two strategies in the steady states or in a more singular and rare case, the alternate dominance of each single strategy (RPS patterns) in the presence of mobility. We simultaneously analyse the effects of the multiplication factor (r), the density of mobile agents in the lattice (ρ) and the mobility parameter (p). It is important to notice that other studies considering mobility in OPGG reveal some features to maintain the cooperation but these results do not explore the existence of cyclic dominance of the strategies (see for example [10]). More precisely, we are interested in how these three important parameters of the game are able to modulate the emergence of spontaneous cooperation, looking simply for coexistence of strategies or cycles of the three possible strategies. In the next section we present the essential points about the model and how our simulations are implemented considering our Monte Carlo Simulations for OPGG with mobility of the players. In Section 3 we present our main results. Finally we summarize our results and present our conclusions in Section 4.

2. The OPGG with Monte Carlo simulations considering mobility of the players

In this paper we consider a population of N players randomly distributed over a square lattice of linear dimension L, with $N \leq L^2$ and the density of players thus defined by $0 \leq \rho = \frac{N}{L^2} \leq 1$. Every player interacts only with its nearest neighbours, and each site in the lattice can contain a player or simply can be a vacancy (empty space). Thereby, if we include the vacancy with status of state, we have a four-state model, where each site can have four possible states: C (cooperator), D (defector), or L (loner) if there is a player occupying the site which represent the three possible strategies for a player and V (vacancy) otherwise. The OPGG with Mobility evolves according to steps of the following algorithm:

1. An agent i is randomly and uniformly chosen among the ρL^2 players;
2. Each cooperator in its neighbourhood contributes to the common pool with a unit of wealth. Defectors participate, but without contribution (free-rider action), while loners stay out of game getting a fixed payoff σ, which we make equal to the unit, without loss of generality.