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Synchronization transmission of spiral wave and turbulence within the uncertain switching network

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HIGHLIGHTS

- Synchronization transmission of spiral wave and turbulence within the uncertain switching network is investigated.
- No matter how many possible topologies of network exist, they can switch from one kind of structure to the other one at any moment.
- The network topologies and the node number do not influence the stability of network synchronization.

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ABSTRACT

We consider the synchronization transmission problem of spiral wave and turbulence within the uncertain switching network. Through constructing reasonably the Lyapunov function of the network, the uncertain switching network not only can transfer synchronously the spiral wave and turbulence originated from the synchronization target, but also the chattering near the synchronization target can be eliminated, which indicates that the synchronization performance of the network is more stable. At the same time, the adaptive laws of the uncertain parameters are designed and the uncertain parameters in the switching network nodes are replaced effectively.

1. Introduction

The spiral wave and turbulence belong to the spatiotemporal patterns caused by nonlinear systems under certain conditions, and those phenomena can often be observed in many systems [1–4]. For instance, many kinds of neurons can show complex nonlinear effects such as spiral wave and turbulence. Since it has been recognized that there is a strong function of information transfer and information processing between the coupled neurons, the investigation about restraint and synchronization transmission of spiral wave and turbulence has aroused comprehensive attention. Subsequently, the various methods have been proposed to control and synchronously transmit the spiral wave and turbulence [5–11].

It is found gradually that many systems engendering spiral wave and turbulence, such neurons, are not isolated, they belong to some kind of assembly constituted by a mass of interconnected systems and such an assembly is referred as network. Among them, those systems are taken as nodes of networks, and most correlations among them are in accord with the characteristics of small-world network [12], scale-free network [13] and switching dynamical network. Subsequently, the restraint and synchronization transmission of various nonlinear effects within complex network have been concerned in recent years. So far, the synchronization which is the most typical collective behavior of complex network has become hot topic and exhibited its unique application potential in comprehensive fields [14–19]. Some theoretical approaches for network synchronization have also been established, for instances, Master stability function method [20], adaptive

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control [21,22], pinning technique [23,24], impulsive control [25], etc. Especially, the synchronization problem of network with delay effect and uncertain parameters has aroused great research interest of scholars at home and abroad [26–28]. The reasons lie in that the delay effect is inevitable in the actual connection of network nodes and signal transmission process among nodes. At the same time, there exist some uncertain parameters in network owing to technology limitation and external turbulence.

It is worth noting that most related works reported previously about the network synchronization mainly focused on the so-called time-domain evolution, that is, the state variable of network node is only time function. However in practice, it can evolve not only with time but also with space. For example, the evolution of spiral wave and turbulence exactly corresponds to spatiotemporal dynamics behaviors. Obviously, it is more tally with actual situation to analyze network synchronization through adopting spatiotemporal system as node of complex network. Compared to time-domain network, however, the spatiotemporal one takes on more intense nonlinear and it is more difficult to control and synchronize because of the existing of space diffusion term within the network, which causes the existing methods about the network control and synchronization are not effective enough. The chattering phenomenon of network node appears even near synchronization target, leading to the unstable performance of network synchronization. Therefore, the synchronization investigations of spatiotemporal network with uncertain parameters are still relatively few due to the some factors discussed above.

Based on above discussions, a novel technique is proposed in our work to investigate the synchronization transmission of spiral wave and turbulence within the uncertain switching network. Lyapunov function of the network is constructed reasonably. Not only the spiral wave and turbulence originated from synchronization target are transmitted synchronously within the uncertain switching network, but also the chattering of network node near the synchronization target is effectively eliminated, which makes the network synchronization performance more stable. At the same time, the adaptive laws of uncertain parameters are designed and the uncertain parameters in the network nodes are perfectly replaced.

2. Models engendering spiral wave and turbulence

We introduce two typical models which can engender spiral wave and turbulence here, and they are taken as the synchronization target and the nodes of switching network with uncertain parameters, respectively.

Firstly, the Fitzhugh–Nagumo system is selected as the synchronization target and its dynamics equation is [29]

$$\begin{cases} \dfrac{\partial u_1(r,t)}{\partial t} = \varepsilon^{-1} u_1(r,t)(1-u_1(r,t))\left(u_1(r,t) - \dfrac{u_2(r,t)+b_1}{a_1}\right) + D_1\nabla^2 u_1(r,t) \\ \dfrac{\partial u_2(r,t)}{\partial t} = \varphi(u_1(r,t)) - u_2(r,t) \end{cases} \tag{1}$$

where $u_1(r,t)$ and $u_2(r,t)$ are state variables and the function $\varphi(u_1(r,t))$ is of following expression form

$$\varphi(u_1(r,t)) = \begin{cases} 0 & (0 \le u_1(r,t) < 1/3) \\ 1 - c_1 u_1(r,t)(u_1(r,t)-1)^2 & (1/3 \le u_1(r,t) \le 1) \\ 1 & (u_1(r,t) > 1) \end{cases} \tag{2}$$

here $r = (x, y)$ indicates that the spatial coordinate variable considered is 2-dimension situation. a_1, b_1, c_1 and ε are parameters and D_1 denotes the diffusion coefficient. $\nabla^2 = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}$ is Laplace operator.

The parameters are chosen as $a_1 = 0.84, b_1 = 0.07$ and $c_1 = 6.75$, besides the diffusion coefficient is set as $D_1 = 1$. Corresponding to the different values of parameter ε, Fitzhugh–Nagumo system can take on very complex and abundant behaviors of spatiotemporal dynamics. The investigation results show that, while parameter ε is in the range of $0 < \varepsilon \le 0.06$, the system will exhibit stable rotating spiral wave under appropriate initial values. When $\varepsilon > 0.06$, however, the system starts to become instable. Once $\varepsilon > 0.07$, the system will go into spatiotemporal chaos, i.e. turbulence. Figs. 1 and 2 are projective patterns of state variable $u_1(r,t)$ under different ε values. Here the system dimensions are taken as 100×100 and time series is set as 100. Fig. 1 shows the system takes on stable rotating spiral wave and Fig. 2 displays the system is in the turbulence state.

Activator–Inhibitor models are taken as nodes to construct an uncertain network. The state equation of Activator–Inhibitor model is as follows [30]

$$\begin{cases} \dfrac{\partial v_1(r,t)}{\partial t} = \dfrac{v_2(r,t) - v_1(r,t)}{(v_2(r,t)-v_1(r,t))^2+1} - m v_2(r,t) + \nabla^2 v_1(r,t) \\ \dfrac{\partial v_2(r,t)}{\partial t} = a_2[b_2 - (v_2(r,t) - v_1(r,t))] + D_2\nabla^2 v_2(r,t) \end{cases} \tag{3}$$

where $m = 0.05$, $a_2 = 0.02$ and $b_2 = 1.25$ are parameters and $D_2 = 8$ is diffusion coefficient. Similarly, $\nabla^2 = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}$ is Laplace operator.

Fig. 3 is the projective pattern of state variable $v_1(r,t)$ with spatiotemporal evolution. Here, the system dimensions are also taken as 100×100. It can be known from Fig. 3 that Activator–Inhibitor model is in turbulence state. Compared with the turbulence state of Fitzhugh–Nagumo system in Fig. 2, the randomicity of turbulence in Fig. 3 is more marked.