Physica A 472 (2017) 77–85

Contents lists available at ScienceDirect

Physica A

journal homepage: www.elsevier.com/locate/physa

# The avalanche process of the fiber bundle model with defect

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## HIGHLIGHTS

- The fiber bundle model with defect is constructed based on the classical fiber bundle model.
- The defect has a significant impact on the mechanical properties of the bundle.
- The statistical properties of the model are still harmonious with the classical fiber bundle model.

## ARTICLE INFO

*Article history:*
Received 16 August 2016
Received in revised form 8 December 2016
Available online 4 January 2017

*Keywords:*
Fiber bundle model
Break dynamic
Avalanche process
Defect

## ABSTRACT

In order to explore the impacts of defect on the tensile fracture process of materials, the fiber bundle model with defect is constructed based on the classical fiber bundle model. In the fiber bundle model with defect, the two key parameters are the mean size and the density of defects. In both uniform and Weibull threshold distributions, the mean size and density all bring impacts on the threshold distribution of fibers. By means of analytical approximation and numerical simulation, we show that the two key parameters of the model have substantial effects on the failure process of the bundle. From macroscopic view, the defect described by the altering of threshold distribution of fibers will has a significant impact on the mechanical properties of the bundle. While in microscopic scale, the statistical properties of the model are still harmonious with the classical fiber bundle model.



## 1. Introduction

The failure phenomena of structures and materials are a considerable complex sets of phenomena in science and technology. The failure of actual materials often cannot be descripted by a simple linear equation for the inherent nonuniformity and disorder in materials. As a result, the theoretical approach of statistical physics is widely used to investigate the properties of the rupture process and their microscopic mechanism. Furthermore, the fluctuation rather than the average property plays a key role in the description of the fracture process [1]. Most statistical investigations on the rupture of disordered materials rely on the fiber bundle model (FBM), which in most cases, can correctly capture the collective static and dynamic properties of fracture failure in loaded materials [2,3]. The algorithm of the FBM is so simple that it is relatively easy to obtain exact results analytically or trustable statistical properties numerically.

In general, the FBM is assumed to be composed of a set of fibers whose break strengths are assumed to comply with a certain statistical law, such as uniform or Weibull distribution. If the load exceeds the threshold value, the fiber will fail. Under stress-controlled loading condition, the bundle is loaded parallel to the fiber direction. After each fiber failure, the

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http://dx.doi.org/10.1016/j.physa.2017.01.017

load carried by the failed fiber is redistributed among the intact fibers. As a result, the subsequent load redistribution can lead to a series of avalanches, which can either stop after a certain number of consecutive failures, keeping the integrity of the bundle, or can be catastrophic, resulting in the macroscopic failure of the entire system. According to the strength of transverse association in the rupture process, the mechanism of the stress redistributed among the intact fibers can be classified into several categories, such as the global load sharing (GLS), the local load sharing (LLS), and so on. In addition, some researches show that stress redistribution in actual heterogeneous materials should fall in many intermediate load sharing forms, such as the power law redistribution rule [4].

In failure process of materials, the most important characteristic of the macroscopic mechanical properties is the stress–strain relationship. In the quasi-statically load increasing, there exists a critical stress $\sigma_c$, which is the maximum stress before the catastrophic failure of the whole system. On the other hand, the statistical properties of the failure process can be intuitively descripted by the size distribution of the burst avalanches, which can be monitored experimentally by acoustic emission techniques [5–7]. In GLS case, the avalanche size distribution of the classical FBM with various fracture threshold distribution follow a power law with a universal exponent $-5/2$ [8–10]. While in LLS case, the avalanche size distribution is more complicated, depending on the specific form of the threshold distribution and the tensile fracture property of a single fiber [11,12].

The classical FBM only considers the identical brittle fibers which is complete linear elastic before the final brittle fracture. In order to accurately describe the fracture process of various disordered materials, some extended FBM are constructed from the following two perspectives: the form of stress redistribution and the tensile fracture property of a single fiber [13]. In the first case, Hidalgo et al. [14] introduced an interpolation form between the global and the local load sharing schemes. By varying the correlation strength between an intact element and the rupture point, the crossover behavior from mean-field approach to short-rang correlation was obtained in the properties of the FBM. Biswas and Chakrabarti [15] proposed a heterogeneous load sharing FBM and showed the critical behavior crosses over from GLS to LLS at some effective site percolation threshold. Pradhan et al. [16] built a FBM with a mixed form of stress redistribution and numerically simulated the crossover behavior between GLS and LLS. Biswas and Sen [17] introduced an efficient redistribution scheme following which the bundle system can carry the maximum load. In order to describe numerous non-brittle fracture process of various biological materials, some complicated tensile fracture properties were introduced to a single fiber instead of the simple brittle fracture. For instance, the continuous damage FBM [18], the continuous damage FBM with strong disorder [19], the FBM with stick–slip dynamics [20–22], and the multilinear FBM [23]. In addition, Some mixed FBM were also introduced to describe a lot of heterogeneous materials. For example, Divakaran et al. [24,25] studied two kinds of FBM with mixed fibers, the one is the mixed fiber bundle with uniform distribution thresholds which can be regarded as the limitation case of random fiber bundle with many discontinuities in the threshold distribution [26,27]; the other is the FBM with two different Weibull distribution [25]. On the other hand, Raischel et al. [28] constructed a plastic FBM and explained that the finite load baring capacity of broken fibers has a substantial effect on the failure process of the bundle. Then, Bosia et al. [29] developed a hierarchical FBM consisting of a certain percentage of brittle fibers and elastic–plastic fibers to simulate the hierarchical structure of some biological materials such as spider silk.

In the research of mechanical property of materials, there is a universal paradox that the experimental strength in actual materials will be smaller than the theoretical strength, especially in brittle materials. One important reason is the existence of defects in actual materials which plays a crucial role in the mechanical behavior of materials under stress, such as the nucleation and propagation of fracture. According to the very different stress–strain response and fracture properties, materials can be broadly classified into three types: brittle, quasi-brittle, ductile and so on. In microscopic scale, the one factor behind these different macroscopic fracture properties is the defect and its kinetic behavior. Furthermore, the defect can be classified into point defect, line defect and planar defect from the geometric scale, which include vacancy, interstitial, impurity, dislocation, microcrack and so on in actual materials [1]. Therefore, it is necessary to specially research for the impacts of defects in materials on its tensile fracture properties.

In this paper, we construct an extended FBM with defect in GLS case. The introduction of defects will alter the threshold distribution of the bundle. In this model, the size and density of defects are the two critical parameters. By analytical approximation and numerical simulation, we reveal the constitutive relationship, the critical stress, the max avalanche size, the avalanche size distribution and the step number of the external load increase as a function of the defect size and the defect density.

## 2. The avalanche process of the FBM with defect in GLS

The FBM with defect is constructed based on the classical FBM. The classical FBM consists of $N$ parallel fibers, all with an identical Young modulus $E_f = 1$ initially. The fibers are generally assumed to crack irreversibly when the stress exceeds a certain threshold. At first, the thresholds of each fiber are assumed to be $\sigma_i$, where $i = 1, 2, \ldots, N$. The threshold $\sigma_i$ of individual fibers is an independent, identically distributed, random variable with a probability density $p$, and a cumulative probability distribution

$$P(\sigma_i) = \int_0^{\sigma_i} p(x)dx. \tag{1}$$