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Boundary induced convection in a collection of polar self-propelled particles

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ABSTRACT

We study a collection of polar self-propelled particles confined to a long two-dimensional channel. We write the coupled hydrodynamic equations of motion for density and polarisation order parameter. At two confined boundaries, density is fixed to the mean and orientation is anti-parallel with fixed magnitude of polarisation. Such boundary conditions make our system similar to a sheared suspension of self-propelled particles, which has many practical applications. Antiparallel alignment at the two confined boundaries and alignment inside the channel create *rolls* of orientation along the long axis of the channel. For zero self-propulsion speed, density and orientation fields are decoupled and density remains homogeneous inside the channel. For finite self-propelled speed, density inhomogeneities develop and these *rolls* move along the long axis of the channel. Density inhomogeneity increases sharply with increasing the self propulsion speed and then reaches a maximum and again decreases for very large speeds. Formation of *rolls* is very similar to the classic problem of Rayleigh–Benard convection in fluid dynamics.

1. Introduction

Collective behaviour of active particles are extensively studied in [1–4]. Large collections of living organisms are known to exhibit highly coherent collective motion [5–9]. This behaviour, often referred to as “flocking” spans an enormous range of length scales and is seen in diverse systems [10–20]. These systems are rigorously studied in *bulk* either (i) using hydrodynamic equations of motion for slow variables (ii) or microscopic rule based models *viz.*: Vicsek’s model [5]. *But* most biological systems are confined to thin geometry [21]. Confinement and boundary plays an important role in variety of problems in biology [21], sheared systems [22] and other places like in fluid dynamics. One classic example include Rayleigh–Benard (RB) convection in fluid [23]. In these confined systems, the effect of boundaries are very important.

Boundary can play very important role in a collection of self-propelled particles. It can induce many interesting phenomena like, in many cases, boundary can induce spontaneous flow inside the channel [24]. We write the phenomenological equations of motion for local density and polarisation order parameter for the collection of polar self-propelled particles Eqs. (1) and (2). Self-propelled speed (SPS) of the particle introduces a non-equilibrium coupling between density and polarisation. For zero SPS both density and polarisation are decoupled. We solve these equations in the confined geometry shown in Fig. 2. At the two boundaries of the channel orientation of rods are antiparallel, which produces a gradient along the confinement direction. Diffusion tries to make them parallel. Hence the competition between above two create *rolls* of

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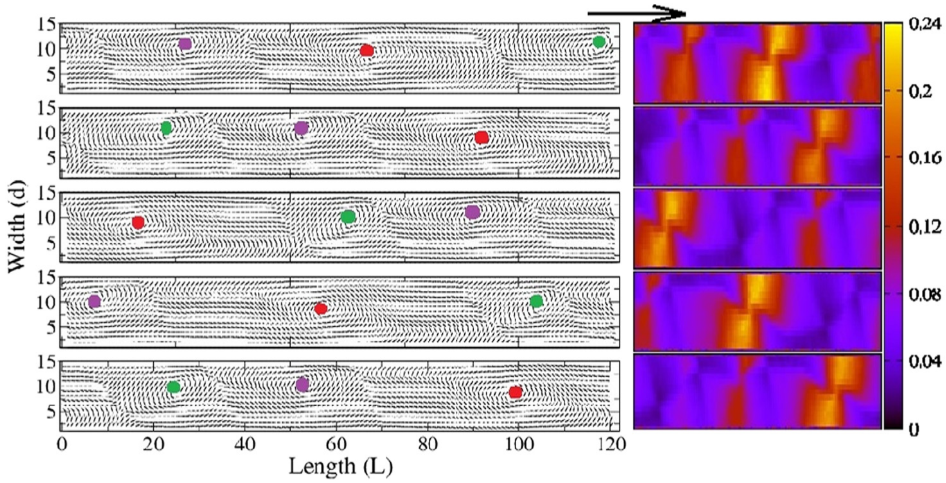

Fig. 1. (Left) Vector plot of local polarisation and (right) density inside the channel for activity $R_A = 0.67$. Different plots are snapshot of polarisation and density at different times. (Left) local polarisation shows vortex type periodic pattern (*rolls*) along the long axis of the channel. Different colour dots on periodic *rolls* represent distinct vortex. Density also shows periodic pattern. Bright regions are high density and dark regions are low density. Top to bottom figures are from small to large time. With time periodic *rolls* for both density and local polarisation moves from one end to other end of the channel. Arrow on the top of the figure represent direction of motion of periodic pattern. This direction is spontaneously chosen from two equally possible direction in the system. (For interpretation of the references to colour in this figure legend, the reader is referred to the web version of this article.)

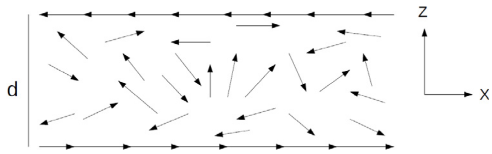

Fig. 2. Geometry of confined channel and orientation of particle at the two confined boundaries. x-direction is chosen along the long axis of the channel and z-direction is the confinement direction. Periodic boundary condition is used along the long axis of the channel. Orientation is parallel to $+x$-direction at bottom boundary ($z = 1$) and parallel to $-x$-direction at top boundary ($z = d$). Magnitude of polarisation $|P| = 1$ is fixed at two boundaries and density is maintained to value $\rho_0 = 0.1$.

orientation along the long-axis of the channel. For zero SPS these rolls are static and density inside the channel is homogeneous. For non-zero SPS both density and polarisation are coupled and such coupling produces moving rolls.

In Fig. 1, we show the (left) vector plot of orientation and (right) density of particles inside the channel for SPS $v_0 = 1.5$ or activity $R_A = 0.67$' at different times. We find inhomogeneous moving pattern of orientation and density along the long axis of the channel Fig. 1 (top to bottom). Arrow indicate the direction of motion.

In rest of the article, Section 2 discusses the model in detail. Here we also write the hydrodynamic equations of motion for density and polarisation. Section 3 discusses the numerical details for solving these equations. We discuss our results in Section 4 and finally conclude with discussion and future aspect of this study in Section 5.

2. Model

We consider a collection of self-propelled particles of length l confined to a two-dimensional channel whose thickness d is very small compare to its long axis L. We fix the length of the channel L and vary the width of the channel $d \ll L$. Orientation at the lower boundary is parallel to horizontal axis and at the upper boundary it is antiparallel and magnitude of polarisation fixed at two boundaries. We also maintain mean density at two confined boundaries to avoid accumulation of particles at boundaries. Periodic boundary condition is used for both density and polarisation along the long axis of the channel. Geometry of confined channel and orientation of particles at the two boundaries is shown in Fig. 2.

2.1. Hydrodynamic equations of motion

Dynamics of the system is described by the equations of motion for hydrodynamic variables for the collection of polar self-propelled particles. We write the phenomenological coupled hydrodynamic equations of motion for density ρ, because total number of particles are conserved and polarisation P, which is an orientation order parameter, is a broken symmetry variable in the ordered state. We write the minimum order terms allowed by symmetry. Two equations are

$$\frac{\partial \rho}{\partial t} = -v_0 \nabla \cdot (\rho \mathbf{P}) + D_\rho \nabla^2 \rho \tag{1}$$