Physica A 477 (2017) 20–33

Contents lists available at ScienceDirect

Physica A

journal homepage: www.elsevier.com/locate/physa

Optimal harvesting policy of a stochastic two-species competitive model with Lévy noise in a polluted environment

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HIGHLIGHTS

- A stochastic competitive model with Lévy noise in a polluted environment is analyzed.
- Both sudden environmental change and toxicant are incorporated.
- Approximation of the optimal harvesting effort and sustainable yields are obtained.
- The Lévy noise and toxicant may affect the optimal harvesting policy significantly.

ARTICLE INFO

Article history:
Received 4 November 2016
Received in revised form 4 January 2017
Available online 22 February 2017

MSC:
34F05
37H10
92B05
60J75

Keywords:
Environmental toxicity
Sudden environmental shocks
Competitive model
Optimal harvesting
Ergodic method

ABSTRACT

As well known that the sudden environmental shocks and toxicant can affect the population dynamics of fish species, a mechanistic understanding of how sudden environmental change and toxicant influence the optimal harvesting policy requires development. This paper presents the optimal harvesting of a stochastic two-species competitive model with Lévy noise in a polluted environment, where the Lévy noise is used to describe the sudden climate change. Due to the discontinuity of the Lévy noise, the classical optimal harvesting methods based on the explicit solution of the corresponding Fokker–Planck equation are invalid. The object of this paper is to fill up this gap and establish the optimal harvesting policy. By using of aggregation and ergodic methods, the approximation of the optimal harvesting effort and maximum expectation of sustainable yields are obtained. Numerical simulations are carried out to support these theoretical results. Our analysis shows that the Lévy noise and the mean stress measure of toxicant in organism may affect the optimal harvesting policy significantly.

1. Introduction

According to the United Nations Food and Agriculture Organization's the State of World Fisheries and Aquaculture report [1]: about 3 billion people depend on fish as their major source of protein. Fish and aquaculture assure the livelihoods of 12 of the world's population, which creates economic benefits of US 2.9 trillion per year. However, as rapid industrial development and human activities, the effect of toxicant on the exposed fish population has become more serious, causing many species survival crisis as toxins destroy the habitat where many fish species need to thrive [2]. Thus, it is meaningful to estimate environmental toxicity so as to develop optimal policies to reduce fishery economic losses, which require

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http://dx.doi.org/10.1016/j.physa.2017.02.019
0378-4371/

qualitative and quantitative estimates of the survival risk of the species in a polluted environment. Recently, the effect of environmental pollution arising from toxic wastes emitted by manufacturing plants or agricultural industry on the fishery resources management has been a interesting topic of considerable researches [3,4].

Pioneered by Clark in 1976 [3], bioeconomic modeling has become an important and powerful tool in analyzing the exploitation policy of natural fishery resources. Various harvesting models have been used to investigate the optimal harvesting policies of renewable resources (see, e.g., Beddington and May [5], Neubert [6], Zhang et al. [4]). In reality, the aquatic ecosystems are inevitable affected by environmental fluctuation [7] (such as, nutrition loading, water temperature variation, etc.), which is an important component for the evolution of the population. Thus, the optimal harvesting models perturbed by the Gaussian white noise have been considered extensively by many authors [8–11]. However, in some cases, these models perturbed by the Gaussian white noise cannot effectively describe the circumstance that the fish population may suffer sudden catastrophic disturbance in nature [12]. The sudden climate change can influence fish population abundance, species distribution and the productivity of marine and freshwater species [12]. For example, Alley [13] pointed out that the abrupt climate change, whether warming or cooling, wetting or drying, could have profound and lasting impacts on natural aquatic ecosystems. Therefore, how to use the discontinuous stochastic process (e.g., Lévy noise [14]) to model these abrupt nature phenomenon in ecosystem is an interesting problem.

In fact, there are various types of random noise perturbations in the real world. Robert et al. [15] pointed out that environmental fluctuation can be classified into two groups: biological fluctuation (such as overharvesting, invasion, and disease), and physical fluctuation (for example, storm, abrupt climate change, volcanic eruption, and forest fire). Meanwhile, there are plenty of empirical statistical results [16] to suggest that the move-step-length frequency distributions for some fish species (e.g., shark, teleosts, sea turtle) exhibit longer steps intermittently distributed within the time series, which may be closely related to the Lévy noise [17].

Recent years, many mathematical researchers and physicists utilized the non-Gaussian Lévy noise to characterize the ecosystem suffered from the physically based environmental disturbance. For example, Cognata et al. [18] considered a L–V system of two competing species subject to multiplicative Lévy noise whose statistics is given by the Lévy α-stable distributions. Bao et al. [19] considered a competitive L–V population dynamics with Lévy jumps, the uniform boundedness of the pth moment and the sample Lyapunov exponent for each component were investigated. Liu and Bai [20] analyzed a stochastic mutualism model with Lévy jumps and harvesting, and established the optimal harvesting effort and the maximum of sustainable yield. The readers may refer to [21–26] for other related studies and the books [14,27] from a bird's-eye view for the non-Gaussian Lévy noise and its application for various biosystem. However, as far as we know, few results of the optimal harvesting of a stochastic two-species competitive model with Lévy noise in a polluted environment have been reported. One possible reason is that it is very difficult to get the explicit solution of the corresponding Fokker–Planck equation, and the other reason is that model (2.5) is a nonautonomous system, we cannot directly prove the asymptotic stability in distribution, thus we need develop some approximation methods to estimate its asymptotic properties. In this paper, we will devote our main attention to the investigation on the following problems of model (2.5).

(**Q1**) How does the sudden environmental shock affect the optimal harvesting policy?

(**Q2**) Which factors influence the expected sustainable yield in a random fluctuating polluted environment?

To answer these questions, the remainder of this paper is organized as follows. The model formulation is given in the next Section. Then, in Section 3, we establish the main results of the optimal harvesting effort and the expectation of sustainable yield of model (2.5). In Section 4, numerical simulations are carried out to support our theoretical results. Finally, a brief discussion is presented in Sections 5.

2. Model formulation

By assuming the environmental disturbance of intrinsic growth rate subjected to a Gaussian white noise, Liu et al. [28] proposed the following two-species stochastic competitive population model in a polluted environment:

$$\begin{cases} dx_1(t) = x_1(t)[\bar{r}_{10} - r_{11}C_0(t) - a_{11}x_1(t) - a_{12}x_2(t)]\,dt + \bar{\sigma}_1 x_1(t)dB_1(t), \\ dx_2(t) = x_2(t)[\bar{r}_{20} - r_{21}C_0(t) - a_{21}x_1(t) - a_{22}x_2(t)]\,dt + \bar{\sigma}_2 x_2(t)dB_2(t), \\ \dfrac{dC_0(t)}{dt} = kC_e(t) - \dfrac{p_1\theta\beta}{k} - (l_1 + l_2)C_0(t), \\ \dfrac{dC_e(t)}{dt} = -hC_e(t) + u(t), \end{cases} \tag{2.1}$$

where $x_i(t)(i = 1, 2)$ is the density of the ith species at time t, $C_0(t)$ is the toxin concentration in the organism at time t, and $C_e(t)$ is the toxin concentration in the environment at time t. All the parameters in model (2.1) are assumed to be positive constants, and the biological meanings are listed in Table 1.