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Critical mass of public goods and its coevolution with cooperation

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HIGHLIGHTS

- The provision of public goods suffers from free riders, which can be described as the 'tragedy of the commons'.
- In this paper, the enhancing parameter was rescaled to a Fermi–Dirac distribution function, and two types of public goods, consumable public goods, and reusable public goods were studied.
- The evidence showed the existence of equilibrium in competition between individual interest and the common interests.
- An intermediate standard believed would be most beneficial to the provision of public goods, and be closely related to the basic group.

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ABSTRACT

In this study, the enhancing parameter represented the value of the public goods to the public in public goods game, and was rescaled to a Fermi–Dirac distribution function of critical mass. Public goods were divided into two categories, consumable and reusable public goods, and their coevolution with cooperative behavior was studied. We observed that for both types of public goods, cooperation was promoted as the enhancing parameter increased when the value of critical mass was not very large. An optimal value of critical mass which led to the best cooperation was identified. We also found that cooperations emerged earlier for reusable public goods, and defections became extinct earlier for the consumable public goods. Moreover, we observed that a moderate depreciation rate for public goods resulted in an optimal cooperation, and this range became wider as the enhancing parameter increased. The noise influence on cooperation was studied, and it was shown that cooperation density varied non-monotonically as noise amplitude increased for reusable public goods, whereas decreased monotonically for consumable public goods. Furthermore, existence of the optimal critical mass was also identified in other three regular networks. Finally, simulation results were utilized to analyze the provision of public goods in detail.

1. Introduction

The provision of public goods suffers from free riders [1–5], which is also referred to as the 'tragedy of the commons' [6]. Since selfishness is inherent to humans, it is understandable that rational individuals will act in accord with their own interests. However, collaboration is extremely important for sustainable development and social stability, and failure to

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cooperate can result in the exploitation of public goods (i.e., environmental resources and social benefits). The public goods game (PGG) is extensively utilized for investigating the persistence of cooperative behaviors during social dilemma [7–13]. In a typical PGG played by N individuals, cooperators contribute a certain amount to the public pool, while defectors contribute nothing. The total contribution is multiplied by an enhancing factor r, and redistributed equally to each participant. Thus, defectors bear none of the cost, yet collect identical benefits as cooperators. This ultimately results in widespread defection in a well-mixed population for $r < N$ [10]. Mechanisms of repeat interactions [14] through, direct and indirect reciprocity [15–19] have been introduced to describe the emergence of cooperation. Punishment and reward are proven effective in promoting cooperation [20–24], but may involve high costs to punish and reward negative and positive members, respectively. To study the persistence of cooperation, spatially structured populations and voluntary participation [25–29] have been utilized as well, and heterogeneity [27,30–33], with respect to sample size of the PGG, individual contribution to the group, and group diversity.

In PGG, r represents the value of public goods to the public. The value is affected by the nature of the public goods and the collaborative behaviors of the public. Previous studies have investigated the depreciation of the value of public goods as a linear function of the number of cooperators [34,35], and have observed that the cooperation density improves by with the introduction of critical mass of moderate value [36–39]. However, in reality, the value of public goods does not depreciate linearly with cooperative behaviors, but instead it possesses hysteresis nature, and changes in a complex manner. In this study, the enhancing parameter was rescaled to a Fermi–Dirac distribution function in which both the inherent nature of public goods and the behaviors of the public are considered. The critical mass was realized as a standard provision of public goods, and parameter Γ determined the depreciation rate of the value of public goods.

This manuscript is structured in the following manner: Section 2 provides a description of the model in detail, Section 3 shows the numerical simulations and the corresponding analysis, and Section 4 lists the conclusions.

2. The model

The public goods game is studied on a square lattice with periodic boundary conditions. Each player on each node only has two alternative strategies, cooperate and defect, and interacts with its four nearest neighbors. Each player together with his nearest neighbors form a basic-interacting group whose scale G is defined as the total number of players in this group. The payoff P_x for each individual is accumulated from the nearest neighbors centered on him, and calculated as follows,

$$P_x = r\frac{n_c \cdot c}{G} - s_x \cdot c, \tag{1}$$

where r is the enhancing parameter indicating the value of the public goods to the public, and n_c is the number of cooperators in a basic group. c is the cost of a cooperator in the public goods game, and set $c = 1$ without losing generality. s_x is the strategy adopted by player x, and $s_x = 1$ if x is a cooperator, and 0 if is a defector.

After each time step, all the players will update their strategies synchronously according to the following rule. A player x randomly selects one nearest neighbor y, and adopts its strategy with a probability $W(s_x \to s_y)$,

$$W(s_x \to s_y) = \frac{1}{1 + \exp\left[\frac{-(P_y - P_x)}{\kappa}\right]}, \tag{2}$$

where κ denotes the amplitude of the noise level, and is set $\kappa = 0.1$ in this study.

Considering that value of public goods r is affected by both the nature of public goods and cooperative behaviors of the public, we rescaled r as,

$$r = \frac{r_0}{1 + \exp\left[\frac{-(n_c - cm)}{\Gamma}\right]}, \tag{3}$$

where r_0 is the full value of public goods to the public, and cm (a non-negative integer, $0 \le cm \le G$) is the critical mass. $\Gamma \ge 0$ influences the depreciation rate of r_0, and a smaller Γ induces a sharper deprecation around $n_c = cm$.

It should be known from Eq. (3) that cm is a standard determining the number of essential contributors for the provision of public goods. Γ will represent different natures of public goods: Reusable public goods ($\Gamma = 0$) which will return the full value of $r = r_0$ once the condition $n_c \ge cm$ is met, and Consumable public goods ($\Gamma > 0$) whose value will decrease as long as they are consumed.

3. Simulation and analysis

Simulations were outperformed using a population of $N = 100 \times 100$ individuals. Initially, cooperators and defectors were randomly distributed with a equal probability 1/2 on the square lattice. The key quantity for characterizing the cooperative behavior is the density of cooperators ρ_c, which was defined as the fraction of cooperators in the whole population. In all simulations, ρ_c was obtained by averaging the last 5000 Monte Carlo time steps of the total 15,000. Each data point represents an average of 50 realizations.