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Stochastic stationary response of a variable-mass system with mass disturbance described by Poisson white noise

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HIGHLIGHTS

- The stationary responses of nonlinear stochastic variable-mass system are studied.
- The mass disturbance has marked influence on system responses.
- Differences between two kinds of mass-disturbance on system responses are explored.
- The theoretical analyses are verified by numerical results.

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ABSTRACT

Variable-mass systems have received widespread attention and show prominent significance with the explosive development of micro- and nanotechnologies, so there is a growing need to study the influences of mass disturbances on systems. This paper is devoted to investigating the stochastic response of a variable-mass system subject to weakly random excitation, in which the mass disturbance is modeled as a Poisson white noise. Firstly, the original system is approximately replaced by the associated conservative system with small disturbance based on the Taylor expansion technique. Then the stationary response of the approximate system is obtained by applying the stochastic averaging method. At last, a representative variable-mass oscillator is worked out to illustrate the effectiveness of the analytical solution by comparing with Monte Carlo simulation. The relative change of mean-square displacement is used to measure the influences of mass disturbance on system responses. Results reveal that the stochastic responses are more sensitive to mass disturbance for some system parameters. It is also found that the influences of Poisson white noise as the mass disturbance on system responses are significantly different from that of Gaussian white noise of the same intensity.

1. Introduction

In stochastic dynamics, there were considerable efforts devoted to the study of invariable-mass systems while the variable-mass systems have not drawn enough attention. In fact, in natural phenomena and engineering technology domains, it is unreasonable for some systems that the mass is considered to be constant [1,2]. For instance, comets loose part of mass due to the gravitational interaction when traveling around the other stars [3,4]. The critical problem-oriented models in engineering fields include the motion of rockets which change mass due to fuel burning [5,6], the motion of jet planes [7], the fluid–structure interaction [8], etc. For those systems with big mass, the expelled or captured mass is relatively

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small and has obvious regularity. Accordingly, mass variation can be described by continuous functions. By far, researches mainly focus on dynamical analyses of deterministic systems with specified mass variation [9], especially with monotone slow-time mass variation [10]. However, in some cases, the mass variation cannot be described with continuous functions due to the existence of unavoidable stochastic mass disturbances [11,12].

In recent years, Microelectromechanical system (MEMS) has attracted enormous attention due to its advantages of small volume, light weight, low energy consumption and intelligence, etc. [13], which has great application prospects and has got brilliant achievements, e.g., miniature motor, micro gyroscope, miniature photoelectric sensors and so on [14,15]. In contrast to the macro mechanical–electrical systems, the scale and mass of MEMS are so small that the adsorption and desorption will significantly change its mass, thereby influencing system dynamic behaviors. For instance, the probe in atomic force microscopy is a crucial component to detect the topographical information of microstructures. Stochastic mass disturbance inevitably reduces its detection precision [16]. In addition, an interesting application of the properties induced by mass variation is detecting the mass of specific molecular [17,18]. Therefore, it is essential to investigate dynamical behaviors of system with mass disturbance in assessing system performance and improving the detection precision. One point worth considering is that the mass variation of MEMS is no longer a small amount when compared with the original mass, which cannot be regarded as a continuous function but discontinuous wave over time. Moreover, due to the random characteristics of adsorption and desorption, mass disturbance should be described by stochastic process.

It is worth noting that the differential equation of motion for stochastic variable-mass systems contain random items related to acceleration, which is different from traditional differential equation in essence. Thus those traditional analysis methods cannot be directly utilized to stochastic variable-mass systems. Proper treatment on the mass disturbance is the key to solve this problem. Recently, the stochastic averaging for quasi-integrable Hamiltonian systems with variable mass and mass disturbances described by Gaussian white noise was proposed by Wang [19]. Zhong et al. investigated the stochastic resonance phenomenon in a fractional oscillator with random mass modeled as a trichotomous noise [20]. To the best of our knowledge, Poisson white noise, as a random pulse, can be considered to simulate the mass disturbance in MEMS. However, the stochastic dynamical behaviors of variable-mass systems with mass disturbance described by Poisson white noise have not been reported yet.

Since the stochastic averaging method of energy envelope was first proposed by Landa and Stratonovich [21], which has been extensively generalized and applied to investigate the stochastic response, stability and bifurcation of single-degree-of-freedom (SDOF) nonlinear systems with light dampings and weakly random excitations [22–27]. Subsequently, based on the Hamiltonian formulation [28], Zhu and his co-workers have further generalized the stochastic averaging method. Especially, in recent years, this stochastic averaging method is developed to study the stochastic response and stability of the nonlinear systems under the excitation of non-Gaussian random processes, such as Poisson white noise [29–32].

Motivated by the above findings, this paper aims to study the stochastic response of a variable-mass system under the external and parametric excitations of Gaussian white noise and small mass disturbance described by Poisson white noise. The paper is organized as follows. A variable-mass system is introduced and replaced by approximate system in Section 2. In Section 3, stochastic averaging method and perturbation method are successively applied to obtain the statistics of stationary response of the approximate system. In Section 4, an example is given in detail to illustrate the validity of the present method. Furthermore, the effects of mass disturbance on stationary responses are explored adequately and the comparison with Gaussian white noise mass perturbation are carried out. Finally, the paper ends with some conclusions in Section 5.

2. System description

Consider a SDOF variable-mass system with mass disturbance modeled as Poisson white noise. The motion is governed by the following equation [19]:

$$\begin{aligned} &m\ddot{x}+\varepsilon^2 c(x,\dot{x})\dot{x}+u(x)=\varepsilon f(x,\dot{x})\xi_e(t),\\ &m=\bar{m}+\varepsilon g\xi_m(t), \end{aligned} \tag{1}$$

in which variable mass m is described by mean mass $\bar{m}$ subjected to small disturbance $\xi_m(t)$; $\bar{m}$ and g are positive constants; ε is a small parameter; $c(x,\dot{x})$ is a differentiable function denoting the damping coefficient; $f(x,\dot{x})$ is an infinite differentiable function representing the amplitude of Gaussian white noise excitation $\xi_e(t)$ which satisfies

$$\langle\xi_e(t)\rangle=0,\qquad \langle\xi_e(t)\xi_e(t+\tau)\rangle=2D\delta(\tau). \tag{2}$$

The mass disturbance $\xi_m(t)$ is Poisson white noise with zero mean, which can be viewed as the formal derivative of the following compound Poisson process $C(t)$:

$$C(t)=\sum_{i=1}^{N(t)}Y_iU(t-T_i), \tag{3}$$

$$\xi_m(t)=\sum_{i=1}^{N(t)}Y_i\delta(t-T_i)=\frac{dC(t)}{dt}, \tag{4}$$