Physica A 473 (2017) 589–602

Contents lists available at ScienceDirect

Physica A

journal homepage: www.elsevier.com/locate/physa

Volume of the steady-state space of financial flows in a monetary stock-flow-consistent model

Aurélien Hazan

Université Paris-Est, France
LISSI, UPEC, 94400 Vitry sur Seine, France

HIGHLIGHTS

- A stationary stock-flow-consistent monetary model of a financial economy is studied.
- The space of flows is the solution of a linear problem with constraints.
- Marginal probability distribution functions are estimated numerically.
- An extension to random connectivity is proposed, linear flows are examined.
- An exponential wealth is found for households.

ARTICLE INFO

Article history:
Received 26 January 2016
Received in revised form 29 November 2016
Available online 10 January 2017

Keywords:
Economics
Physics and society
Constraint satisfaction
Monte-Carlo
Random network
Convex polytope
Finance

ABSTRACT

We show that a steady-state stock-flow consistent macro-economic model can be represented as a Constraint Satisfaction Problem (CSP). The set of solutions is a polytope, which volume depends on the constraints applied and reveals the potential fragility of the economic circuit, with no need to study the dynamics. Several methods to compute the volume are compared, inspired by operations research methods and the analysis of metabolic networks, both exact and approximate. We also introduce a random transaction matrix, and study the particular case of linear flows with respect to money stocks.

In this article we propose an approach to macro-economic modeling inspired by stock-flow consistent (SFC) models [1] and statistical physics, solving a Constraint Satisfaction Problem (CSP) in a way similar to recent works in the field of metabolic networks [2]. The SFC framework provides accounting identities ensuring that "everything comes from somewhere and everything goes some where" [1, p. 38], thanks to budget constraints and behavioral constraints. The formalism of DSGE (Dynamic Stochastic General Equilibrium) is dominant today in macro-economics, partly because the corresponding models can be written in the form of state-space models and estimated in a well-studied statistical framework.[1] Their usefulness has been widely debated among economists [4,5] and physicists [6] because of their inability to predict crises. Many of their hypotheses have been criticized, such as representative rational agents, exogeneity of financial factors, clearing markets where offer always meet demand, etc.… Moreover, DSGE models usually do not implement SFC accounting identities.

E-mail address: aurelien.hazan@u-pec.fr.

[1] See [3, §3.2] for a discussion.

http://dx.doi.org/10.1016/j.physa.2017.01.050
0378-4371/

Most SFC works take place at the macroeconomic aggregate level. Various assets (loans, equities, bonds, ...) and sectors (households, firms, banks, states,...) have been considered in the literature [7]. Models can be more or less detailed, depending on the focus of the study (for example, the production sector can be aggregated or multi-sectoral). The issue of the micro-foundations of SFC models has been tackled with the combination of SFC and agent-based models (ABM). ABM [8,9] can represent large populations of heterogeneous agents, to explore the influence of networks effects, coordination, bounded rationality and learning. However, they do not usually implement stock-flow consistency. Recent works combine SFC and ABM [10–12], providing micro-foundations to SFC models, and imposing macro constraints to ABMs. Nevertheless, the computational cost of ABM simulations is high, and theoretical understanding is limited so far. Calibration and validation are known to be difficult problems.

We consider a simplified stock-flow consistent model developed by macro-economists, where the state of the economy is the set of all stocks and flows of money. It is shown that one can compute the set of admissible steady-state configurations of this simple model. In this steady-state solution space, all configurations are equally weighted, thus allowing unusual states of the financial flows to be encompassed. The marginal probabilities of individual configurations can be approximated over the whole solution space.

Our standpoint is to transpose ideas from the field of metabolic networks where steady-state fluxes have been studied as CSP. These studies were in turn inspired by Von Neumann's growth model of production economies [13]. The results obtained with metabolic networks were successfully compared to experimental data, as in the Red Blood Cell metabolism or the central metabolism of *Escherichia coli* [14,2,15,16]. Such system-scale studies reveal some interesting features of metabolisms, for example the cooperation between pathways. It has been shown also that organisms such as *E. coli* do not necessarily optimize their metabolic fluxes.

The steady-state equilibrium hypothesis is accepted in the field of metabolic networks because of the separation of timescales between metabolic and genetic regulations [17,18]. In economy, the existence of cycles and their corresponding time constants has been the subject of many theories and debates. Recent empirical works are able to identify the timescale at which some specific phenomena operate [19]. In the case of the model examined in this article, we consider that at the time scale that separates two balance sheets (one year), capital accumulation and output growth are slow and will be considered constant (as noted in [20], the global annual per capita growth rate of production is 0.8% on average on the 1700–2012 time interval).

The expected benefits of applying these methods in macro-economics include the analysis of fragilities, notably the sensitivity to arbitrary flow constraints, such as shortages. Indeed, the volume of the solution space evoked above is immediately impacted when constraints are added or removed, and can reveal the flexibility or rigidity of financial flows subject to perturbations.

In Section 1, we detail the model of financial flows that will be used as a benchmark, and present its background from a macro-economic modeling point of view. Comparisons are made with ABM and econophysics. We present the different methods used to compute exactly and approximately the volume of the steady-state solution space. Then in Section 2, the experimental results are explained. Finally, Sections 3 and 4 are devoted to discussion and conclusion.

1. Background and methods

1.1. Steady-state solution space in a stock-flow-consistent model

In SFC models agents are grouped by sectors (banks, firms, workers, state, central bank) that are linked by money transfers. For example:

- Banks lend money to firms, which pay interests to the former.
- Banks pay interests on deposits made by workers.
- Firms pay wages to workers.
- Workers buy consumption goods to firms.
- Firms invest in capital goods bought from other firms.

Assets and liabilities at a given instant t in time are summarized in a balance sheet, where positive and negative signs stand for uses and sources of money. The balance sheet in Table 1 corresponds to the BMW model, discussed in [1, chap. 7], which will be used in this article. In the BMW model, bank issue loans to finance the investments of the productive sector, while households are both consumers and workers. The state and central bank are omitted. Production firms and the banks make no net profit. The net worth of these sectors is zero. The net wealth equals the total tangible capital K.

The transaction matrix sums up all the flows of funds between sectors within a time interval $[t, t + \Delta t]$. Positive and negative signs stand for inflows and outflows of money. They are balanced using a double-entry book-keeping representation where rows sum to zero since each transaction has a counterparty, and columns sum to zero because of the sector's budget constraints.

Table 2 shows the transaction matrix corresponding to the BMW model with one agent per sector. After [1, chap. 7], we make the hypothesis that demand terms, with the subscript d, equal supply terms, with the subscript s. Notations are summarized in Table 3.