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Fluid-structure interactions using different mesh motion techniques

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ABSTRACT

In this work, we compare different mesh moving techniques for monolithically-coupled fluid-structure interactions in arbitrary Lagrangian–Eulerian coordinates. The mesh movement is realized by solving an additional partial differential equation of harmonic, linear-elastic, or biharmonic type. We examine an implementation of time discretization that is designed with finite differences. Spatial discretization is based on a Galerkin finite element method. To solve the resulting discrete nonlinear systems, a Newton method with exact Jacobian matrix is used. Our results show that the biharmonic model produces the smoothest meshes but has increased computational cost compared to the other two approaches.

1. Introduction

Fluid-structure interactions are of great importance in many real-life applications, such as industrial processes, aero-elasticity, and bio-mechanics. More specifically, fluid-structure interactions are important to measuring the flow around elastic structures, the flutter analysis of airplanes [1], blood flow in the cardiovascular system, and the dynamics of heart valves (hemodynamics) [2,3].

Typically, fluid and structure are given in different coordinate systems making a common solution approach challenging. Fluid flows are given in Eulerian coordinates whereas the structure is treated in a Lagrangian framework. We use a monolithic approach (Fig. 1), where all equations are solved simultaneously. Here, the interface conditions, the continuity of velocity and the normal stresses, are automatically achieved. The coupling leads to additional nonlinear behavior of the overall system.

Using a monolithic formulation is motivated by upcoming investigations of gradient based optimization methods [4], and for rigorous goal oriented error estimation and mesh adaptation [5], where a coupled monolithic variational formulation is an inevitable prerequisite.

For fluid-structure interaction based on the 'arbitrary Lagrangian–Eulerian' framework (ALE), the choice of appropriate fluid mesh movement is important. In general, an additional elasticity equation is solved. For moderate deformations, one can pose an auxiliary Laplace problem that is known as harmonic mesh motion [6,7]. More advanced equations from linear elasticity are also available [8,9]. For a partitioned fluid-structure interaction scheme, a comparison was made between different models [10]. The pseudo-material parameters in both approaches were used to control the mesh deformation. If the parameters do not depend on mesh position and geometrical information, both approaches can only deal with moderate fluid mesh deformations. This problem is resolved by using mesh-position dependent material parameters that are used to increase the stiffness of cells near the interface [8]. There are several techniques for choosing these parameters to retain an optimal mesh, such as a Jacobian-based stiffening power [11] that is eventually governed by appropriate re-meshing techniques. We use an ad hoc approach for these parameters, measuring the distance to the elastic structure and adapting the parameters to prevent mesh cell distortion as long as possible.

Here, we also use (for mesh moving) the biharmonic equation that others have studied for fluid flows in ALE coordinates [12]. It was also shown there, that using the biharmonic model provides greater freedom in the choice of boundary and interface conditions. In general, the biharmonic mesh motion model leads to a smoother mesh (and larger deformations of the structure) compared to the mesh motion models based on second order partial differential equations. Larger deformations and structure touching the wall are only possible with a fully Eulerian approach [6,7,13] or in the ALE framework with a full or partial re-meshing of the mesh, i.e., generating a new set of mesh cells and sometimes also a new set of nodes.

Although, the mesh behavior of the harmonic and the biharmonic mesh motion models were analyzed in [12] for different applications, we upgrade these concepts to fluid-structure interaction problems. Moreover, we provide quantitative comparisons of the three mesh motion models.

In the discretization section, we address aspects of the implementation of a temporal discretization, that is based on finite

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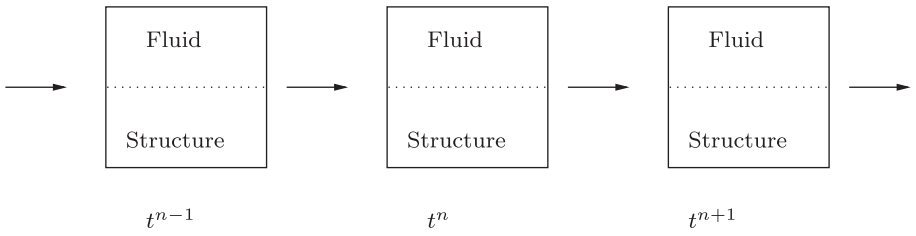

Fig. 1. The monolithic solution approach for fluid-structure interaction.

differences. In particular, we present the one step-θ schemes [14] and the Fractional step-θ scheme [15] in ALE fashion for the monolithic problem. Space discretization is done using a standard Galerkin finite element approach. The solution of the discretized system can be achieved with a Newton method, which is very attractive because it provides robust and rapid convergence. The Jacobian matrix is derived by exact linearization which is demonstrated by an example. Because the development of iterative linear solvers is difficult for fully coupled problems (however, suggestions have been made [16,17]), and we are only interested in solving problems for a low amount of unknowns, we use a direct solver to solve the linear systems.

The outline of this paper is as follows. In the second section, the fluid equations in artificial coordinates, and structure equations for two different material models, are introduced. After, the mixed formulation of the biharmonic equation is introduced for two kinds of boundary conditions. Finally, fluid-structure interaction based on a closed variational setting is proposed. Section 3 presents discretization in time and space of the fluid-structure interaction problems. Moreover, the nonlinear problem is examined through an exact computation of the Jacobian matrix. The computation of the directional derivatives is shown. In Section 4, numerical tests for four problems (in both two and three dimensions) are performed, showing the advantages and the differences between the three mesh motion models. The computations are performed using the finite element software package deal.II [18].

2. Equations

In this section, we briefly introduce the basic notation and the equations describing both the fluid (in the ALE-transformed coordinate system) and structure (in its natural Lagrangian coordinates). Then, we present the monolithic setting for the coupled problem.

2.1. Notation

We denote by $\Omega \subset \mathbb{R}^d$, $d = 2, 3$, the domain of the fluid-structure interaction problem. This domain is supposed to be time independent but consists of two time dependent subdomains $\Omega_f(t)$ and $\Omega_s(t)$. The interface between both domain is denoted by $\Gamma_i(t) = \partial\Omega_f(t) \cap \partial\Omega_s(t)$. The initial (or later reference) domains are denoted by $\widehat{\Omega}_f$ and $\widehat{\Omega}_s$, respectively, with the interface $\widehat{\Gamma}_i$. Further, we denote the outer boundary with $\partial\widehat{\Omega} = \widehat{\Gamma} = \widehat{\Gamma}_D \cup \widehat{\Gamma}_N$ where $\widehat{\Gamma}_D$and $\widehat{\Gamma}_N$denote Dirichlet and Neumann boundaries, respectively. We adopt standard notation for the usual Lebesgue and Soboley spaces and their extensions by means of the Bochner integral for time dependent problems [19]. We use the notation $(\cdot,\cdot)_X$ for a scalar product on a Hilbert space X and $\langle\cdot,\cdot\rangle_{\partial X}$ for the scalar product on the boundary ∂X. For the time dependent functions on a time interval I, the Sobolev spaces are defined by $\mathcal{X} := L^2(I;X)$. Concretely, we use $\mathcal{L} := L^2(I;L^2(\Omega))$ and $\mathcal{V} := H^1(I;H^1(\Omega)) = \{v \in L^2(I;H^1(\Omega)) : \partial_t v \in L^2(I;H^1(\Omega))\}$.

2.2. Fluid in artificial coordinates

Let $\widehat{\mathcal{A}}_f(\hat{x},t) : \hat{\Omega}_f \times I_t \to \Omega_f(t)$ be a piecewise continuously differentiable invertible mapping. We define the physical unknowns $\hat{v}_f$ and $\hat{p}_f$ in $\widehat{\Omega}_f$ by

$$\hat{v}_f(\hat{x},t) = v_f(x,t) = v_f(\widehat{\mathcal{A}}_f(\hat{x},t),t),$$
$$\hat{p}_f(\hat{x},t) = p_f(x,t) = p_f(\widehat{\mathcal{A}}_f(\hat{x},t),t).$$

Then, with

$$\widehat{F}_f := \hat{\nabla}\widehat{\mathcal{A}}_f, \quad \widehat{J}_f := \det\widehat{F}_f,$$

we get the relations [20]:

$$\nabla v_f = \hat{\nabla}\hat{v}_f\widehat{F}_f^{-1}, \quad \partial_t v_f = \partial_t\hat{v}_f - (\widehat{F}_f^{-1}\partial_t\widehat{\mathcal{A}}_f\cdot\hat{\nabla})\hat{v}_f,$$
$$\int_{\Omega_f} f(x)\mathrm{d}x = \int_{\widehat{\Omega}_f}\hat{f}(\hat{x})\widehat{J}\mathrm{d}\hat{x}.$$

With help of these relations, we can formulate the Navier–Stokes equations in artificial coordinates:

Problem 2.1. (Variational fluid problem, ALE framework) Find $\{\hat{v}_f,\hat{p}_f\} \in \{\hat{v}_f^D + \widehat{\mathcal{V}}\} \times \widehat{\mathcal{L}}_f$, such that $\hat{v}_f(0) = \hat{v}_f^0$, for almost all time steps t, and

$$(\widehat{J}_f\hat{\rho}_f(\partial_t\hat{v}_f + (\widehat{F}_f^{-1}(\hat{v}_f - \partial_t\widehat{\mathcal{A}}_f)\cdot\hat{\nabla})\hat{v}_f),\hat{\psi}^v)_{\widehat{\Omega}_f} + (\widehat{J}_f\hat{\sigma}_f\widehat{F}_f^{-T},\hat{\nabla}\hat{\psi}^v)_{\widehat{\Omega}_f}$$
$$- \langle\hat{g}_f,\hat{\psi}^v\rangle_{\widehat{\Gamma}_i\cup\widehat{\Gamma}_N} = 0 \;\forall\hat{\psi}^v \in \widehat{V}_f,$$

$$(\widehat{\mathrm{div}}(\widehat{J}_f\widehat{F}_f^{-1}\hat{v}_f),\hat{\psi}^p)_{\widehat{\Omega}_f} = 0 \;\forall\hat{\psi}^p \in \widehat{L}_f,$$

with the transformed Cauchy stress tensor

$$\hat{\sigma}_f := -\hat{p}_f I + \hat{\rho}_f\nu_f(\hat{\nabla}\hat{v}_f\widehat{F}^{-1} + \widehat{F}^{-T}\hat{\nabla}\hat{v}_f^T).$$

The viscosity and the density of the fluid are denoted by ν_f and $\hat{\rho}_f$, respectively. The function $\hat{g}_f$ represents Neumann boundary conditions for both physical boundaries (e.g., stress zero at outflow boundary), and normal stresses on $\widehat{\Gamma}_i$. Later, this boundary represents the interface between the fluid and structure. We note that the specific choice of the transformation $\widehat{\mathcal{A}}_f$ is up to now arbitrary and left open.

2.3. Structure in Lagrangian coordinates

Usually, structural problems are formulated in Lagrangian coordinates, which means to find a mapping from the physical domain $\Omega_s(t)$ to the reference domain $\widehat{\Omega}_s$. The transformation $\widehat{\mathcal{A}}_s(t) : \widehat{\Omega}_s \times I_t \to \Omega_s(t)$ is naturally given by the deformation itself:

$$\widehat{\mathcal{A}}_s(\hat{x},t) = \hat{x} + \hat{u}_s(\hat{x},t), \quad \widehat{F}_s := \hat{\nabla}\widehat{\mathcal{A}}_s = I + \hat{\nabla}\hat{u}_s, \quad \widehat{J}_s := \det(\widehat{F}_s). \tag{1}$$