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Stability of an SAIRS alcoholism model on scale-free networks☆

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HIGHLIGHTS

- A new alcoholism model with birth and death on complex heterogeneous networks are investigated.
- The spread of alcoholism threshold R_0 is calculated by the next generation matrix method.
- The dynamics of our model are studied.
- The modified SAIRS alcoholism model on weighted contact network is introduced and studied.
- Our results show that it is very important to treat alcoholics to control the spread of the alcoholism.

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ABSTRACT

A new SAIRS alcoholism model with birth and death on complex heterogeneous networks is proposed. The total population of our model is partitioned into four compartments: the susceptible individual, the light problem alcoholic, the heavy problem alcoholic and the recovered individual. The spread of alcoholism threshold R_0 is calculated by the next generation matrix method. When $R_0 < 1$, the alcohol free equilibrium is globally asymptotically stable, then the alcoholics will disappear. When $R_0 > 1$, the alcoholism equilibrium is global attractivity, then the number of alcoholics will remain stable and alcoholism will become endemic. Furthermore, the modified SAIRS alcoholism model on weighted contact network is introduced. Dynamical behavior of the modified model is also studied. Numerical simulations are also presented to verify and extend theoretical results. Our results show that it is very important to treat alcoholics to control the spread of the alcoholism.

1. Introduction

Alcoholism problem of young people has become a significant public health concern. US surveys indicate that about 90% of college students have consumed alcohol at least once [1], and more than 40% of college students have engaged in alcoholism [2,3]. The behaviors of young alcoholics often cause a range of negative consequences. Long-term alcoholism produce negative changes in the brain, such as tolerance and physical dependence. Alcoholism damages almost all parts of the human body and contributes to a number of human diseases including liver cirrhosis, pancreatitis, heart disease, sexual dysfunction and eventually death [4]. Studying of alcoholism have attracted the attention of many scholars and researchers,

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recently. Straus and Bacon [5] began to study college students' alcohol. Sanchez et al. [6] described how alcoholics spread the drinking problems through social contact between people with different drinking habits. Manthey et al. [7] built an epidemiological model to capture the dynamics of campus drinking and to study how the "disease" of drinking was spread on campus. Benedict [8] proposed an SIR-type model and discussed the existence and stability of equilibria. Huo and Song [9] introduced a two-stage model for binge drinking problem, the youths with alcohol problems were divided into those who admit the problem and those who do not admit it. Wang et al. [10] presented a deterministic SATQ-type mathematical model for the spread of alcoholism, with two control strategies to gain insights into the increasingly concerned about health and social phenomenon, some properties of the solutions including positivity, existence and stability were analyzed. For the other mathematical models for alcoholism or smoking, we refer to [11–15] and the references cited there in.

However, the above mentioned alcohol models are all based on the assumption of homogeneous mixing, which means that all individuals mix uniformly and all hosts have identical contact rate. In fact, the contact process of different people may be entirely different in per unit of time. This obviously leads to a new theory that all of such individuals formed a complex network. In contrast to classical compartment models, we consider the whole population and their contacts on complex networks. On complex networks, the nodes represent individuals and the links represent various interactions among those individuals. In other words, networks can be characterized by the connectivity of their nodes. The connectivity of a node is degree, represent by k, which is defined as the number of the links connected to the node. The degree distribution of a network is defined as the probability of a randomly chosen node to have a degree k, represents by $P(k)$. Many networks such as the social networks, the Internet and the World Wide Web have been found to be scale-free networks, that is to say the degree distribution follows a power law distribution $P(k) = ck^{-\gamma}$, $(2 < \gamma \leq 3)$, c is any constant satisfy the equation $\sum_{k=1}^{n} P(k) = 1$. Many epidemic models have been studied follow a power law distribution on complex networks [16–30].

Motivated by the above, we set up an alcoholism model on complex networks to study the propagation dynamics of alcoholism in this paper. Comparing with the above models, we divide alcoholics into light problem alcoholics and heavy problem alcoholics and furthermore take into account individual's birth and death on scale-free complex networks. The paper is organized as follows: The model is formulated in Section 2. The basic reproduction number and existence of alcoholism equilibrium are calculated in Section 3. In Section 4, we analysis the stability of the equilibria. In Section 5, the modified SAIRS alcoholism model on weighted contact network is also introduced. Dynamical behavior of the modified model is studied. In Section 6, sensitivity analysis and numerical simulations are illustrated. In Section 7, we give some conclusions and discussions.

2. Model formulation

From research group of treatment and rehabilitation of the US National Institute on Alcohol Abuse and Alcoholism (NIAAA), in the United States, one "standard" drink contains roughly 14 g of pure alcohol, which is found in: 12 ounces of regular beer, which is usually about 5% alcohol; 5 ounces of wine, which is typically about 12% alcohol; 1.5 ounces of distilled spirits, which is about 40% alcohol. The daily alcohol consumption for men is no more than four standard drinks, women are no more than three standard drinks. The ceiling of "low-risk" alcohol consumption per week is 14 standard drinks for men, and 7 standard drinks for women [30]. If a person whose alcohol consumption is more than daily or weekly drinking ceiling, he/she most likely develop to "abuse alcohol" or "addicted alcohol". In this paper, our model is based on dividing the whole population into four compartments, namely: the susceptibles, refer to the people who do not drink or drink only moderately, denoted by $S(t)$; the light problem alcoholics, denoted by $A(t)$, refer to the drinkers who drink beyond daily or weekly ceiling and drink 4 to 5 standard drinks per day; the heavy problem alcoholics, denoted by $I(t)$, refer to the drinkers who drink more than daily and weekly limits and drink more than 5 standard drinks per day; the recovered, refer to the people who recover from alcoholism after treatment, denoted by $R(t)$. The total number of population at time t is given by

$$N(t) = S(t) + A(t) + I(t) + R(t). \tag{2.1}$$

We assume that individuals are spatially distributed on complex network N. Each node of N is either vacant or occupied by one individual. Motivated by [21], we give each site a number: 0, 1, 2, 3 or 4, respectively. we interpret the five states: state 0: vacant; state 1: a susceptible individual occupied; state 2: a light problem alcoholic occupied; state 3: a heavy problem alcoholic occupied; state 4: a recovered individual occupied. The states of the system at time t can be described by a set of numbers $\{0, 1, 2, 3, 4\}$. That means if the system is in state A and the site $x \in N$, then $A_t(x) \in \{0, 1, 2, 3, 4\}$. Each site can change its state with a certain rate. An empty site can give new individuals to susceptible compartment at rate b. Susceptible individuals can be infected by contact with light problem alcoholics at the rate ρ_1, or be infected by contact with heavy problem alcoholics at the rate ρ_2, we assume that if a susceptible individual infected by alcoholics, he will enter into light problem alcoholics compartment at rate ρ, where $\rho = \rho_1\theta_1 + \rho_2\theta_2$, and $\theta_1(\theta_2)$ denotes the probability of a susceptible individual contacts with a light problem alcoholic (heavy problem alcoholic). A light problem alcoholic becomes a heavy problem alcoholic at rate $\alpha\beta$ due to excessive drinking. Here, α represents the removal rate from light problem alcoholics compartment and parameter β is the proportion of light problem alcoholics that turn into heavy problem alcoholics. A light problem alcoholic can recover due to treatment at rate $\alpha(1-\beta)$. A heavy problem alcoholic becomes a recovered individual at rate γ due to treatment. A recuperator individual becomes a susceptible at rate σ. We assume that the death rate of all individuals is μ. If an individual dies, the corresponding side will become empty. The light problem alcoholics are the source of the heavy problem alcoholics in our model. So if light problem alcoholics extinct, there is no the heavy