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Q1 Characterization in bi-parameter space of a non-ideal oscillator

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HIGHLIGHTS

- We have studied the dynamics of a non-ideal Duffing oscillator.
- We identified new features on Duffing oscillator parameter space.
- Our results show organized distribution of periodic windows.
- We observed intertwined basins of attraction for coexisting multiple attractors.

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ABSTRACT

We investigate the dynamical behavior of a non-ideal Duffing oscillator, a system composed of a mass–spring–pendulum driven by a DC motor with limited power supply. To identify new features on Duffing oscillator parameter space due to the limited power supply, we provide an extensive numerical characterization in the bi-parameter space by using Lyapunov exponents. Following this procedure, we identify remarkable new organized distribution of periodic windows, the ones known as Arnold tongues and also shrimp-shaped structures. In addition, we also identify intertwined basins of attraction for coexisting multiple attractors connected with tongues.

1. Introduction

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In recent years, there has been an increasing amount of work on nonlinear dynamics characterizing the possible structures in two-dimensional control parameter (bi-parameter) space [1]. Accordingly, periodic windows with important features, mainly shrimp-shaped structures [2] and Arnold tongues [3–5], have been identified in several systems such as two-gene model [6], impact oscillator [7,8], dissipative model of relativistic particles [9], tumor growth model [10], Chua's circuit [11–13], prey–predator model [14], and Red Grouse population model [15].

In the nonlinear dynamics context, oscillators with mechanical coupling have recently attracted a significant attention due to the complexity of the dynamics for high degree-of-freedom devices and possible applications to advanced technologies [16–20]. Among the class of mechanical coupling oscillators, an interesting example is the mass–spring–pendulum

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system [21,22]. Svoboda and collaborators studied a system of masses with a pendulum, where the pendulum is attached to one mass of a chain of masses connected by springs [23]. They showed that autoparametric resonance can arise. In Ref. [24] was investigated the influence of nonlinear spring on the autoparametric system. It was verified the existence of rich dynamics such as chaotic oscillations.

In this work, we investigate the parameter space organization of a non-ideal Duffing oscillator, namely, the mass–spring–pendulum system. Duffing oscillator is a forced oscillator with a nonlinear elasticity, and it is described by a nonlinear differential equation of second-order that has been used in a variety of physical processes. This oscillator is well known in engineering science, and it has been used to model the dynamics of types of electrical and mechanical systems. Almong and collaborators experimentally studied signal amplification in a nanomechanical Duffing resonator via stochastic resonance [25]. The Duffing oscillator is also a useful model to study the dynamics behavior of structural systems, such as columns, gyroscopes, and bridges [26].

The non-ideal character of the studied oscillator is a consequence of the fact that the source of energy is given by a DC motor with limited power supply [27,28]. Previous studies of this system have shown a rich dynamical behavior with several nonlinear phenomena, like quasi-periodic attractors, chaotic regimes, crises, coexistence of attractors, and fractal basin boundaries [29–31]. Here, our main purpose is to provide a global parameter analysis of the behavior of this oscillator with a mechanical coupling. The main features found in the parameter space were the self-similar structures, such as shrimps and Arnold tongues. Comparing with results from parameter spaces of ideal oscillators, these Arnold tongue attributes are a consequence of the non-ideal character of this oscillator.

This paper is organized as follows. In Section 2, we present the mathematical description of the non-ideal Duffing oscillator. In Section 3, we provide characterization of the periodic windows identified in the bi-parameter space. In Section 4, we also provide an example of a possible coexistence of multiple attractors and their corresponding basins of attraction. The last section contains our main conclusions.

2. Non-ideal Duffing oscillator

Several mechanical systems can be described by the Duffing equation. Tusset and Balthazar [32] studied ideal and non-ideal Duffing oscillator with chaotic behavior. They suppressed the chaotic oscillations through the application of two control signals. In this work, we consider a non-ideal system consisting of a mass, spring and pendulum. Fig. 1 shows a schematic model of the non-ideal oscillator [31], that is composed of a cart (mass M), with a pendulum (mass m and length r), connected to a fixed frame by a nonlinear spring and a dash-pot. We denote by X the displacement of the cart and by φ the angular displacement of the pendulum.

The equations of motion, obtained by using Lagrangian approach, for both the cart and the pendulum are given by

$$(m+M)\frac{\mathrm{d}^2X}{\mathrm{d}t^2}+c_1\frac{\mathrm{d}X}{\mathrm{d}t}-k_1X+k_2X^3=mr\left(\frac{\mathrm{d}\varphi}{\mathrm{d}t}^2\sin\varphi-\frac{\mathrm{d}^2\varphi}{\mathrm{d}t^2}\cos\varphi\right), \tag{1}$$

$$mr^2\frac{\mathrm{d}^2\varphi}{\mathrm{d}t^2}+c_2\frac{\mathrm{d}\varphi}{\mathrm{d}t}+mgr\sin\varphi=E-mr\frac{\mathrm{d}^2X}{\mathrm{d}t^2}\cos\varphi, \tag{2}$$

where E is a constant source of energy. According to Eq. (1), for $k_1 < 0$, the Duffing oscillator can be interpreted as a forced oscillator with a spring whose restoring force is $F = k_1X - k_2X^3$. Whereas, for $k_1 > 0$, the Duffing oscillator describes the dynamics of a point mass in a double well potential, such as a deflection structure building model.

Considering $x \equiv X/r$ and $\tau \equiv \omega_1 t$ ($\omega_1 \equiv \sqrt{\frac{k_1}{m+M}}$), the equations of motion are rewritten in the following form:

$$\ddot{x}+\beta_1\dot{x}-x+\gamma x^3=\varepsilon\left(\dot{\varphi}^2\sin\varphi-\ddot{\varphi}\cos\varphi\right), \tag{3}$$

$$\ddot{\varphi}+\beta_2\dot{\varphi}+\Omega^2\sin\varphi=\alpha-\ddot{x}\cos\varphi \tag{4}$$

for $\beta_1 \equiv \frac{c_1}{(m+M)\omega_1}$, $\gamma \equiv \frac{k_2}{k_1}r^2$, $\varepsilon \equiv \frac{m}{m+M}$, $\beta_2 \equiv \frac{c_2}{mr^2\omega_1{}^2}$, $\Omega \equiv \frac{\omega_2}{\omega_1}$ ($\omega_2 \equiv \sqrt{g/r}$), and $\alpha \equiv \frac{E}{mr^2\omega_1{}^2}$ (source of energy).

These equations of motion correspond to a simplified mathematical model for oscillator with a limited power supply. In this case, the source of energy is given by a DC motor and the parameter α is associated with its input voltage.

3. Arnold tongues and shrimps

In this section, we present numerical results identifying periodic windows in bi-parameter space for the non-ideal Duffing oscillator. The simulations were performed by using the fourth-order Runge–Kutta method with a fixed step. The control parameters were fixed at $\beta_1 = 0.05$, $\beta_2 = 1.5$, $\gamma = 0.1$, and $\Omega = 1.0$. We consider for dynamic investigations the variations of parameters ε (the ratio of the masses) and α (input voltage of the DC motor).

First, we use a bifurcation diagram, as shown in Fig. 2(a) and (b) for $\varepsilon = 0.09$, to verify possible solutions generated by the oscillator. This diagram is constructed varying the control parameter α. For each value of the parameter, we plot the local maximum values of the dynamical variable x neglecting the transients. As can be seen in Fig. 2(b), the bifurcation diagram is