



Stationary distribution and extinction of a stochastic SIRS epidemic model with standard incidence

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HIGHLIGHTS

- A stochastic SIRS epidemic model with standard incidence is proposed and studied.
- Establish sufficient conditions for the existence of ergodic stationary distribution.
- We also establish sufficient conditions for extinction of the disease.

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ABSTRACT

In this paper, we consider a stochastic SIRS epidemic model with standard incidence. By constructing suitable stochastic Lyapunov function, we establish sufficient conditions for the existence of ergodic stationary distribution of the model. Moreover, we also establish sufficient conditions for extinction of the disease.

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1. Introduction

Recently, mathematical models have played an important role in describing the characteristics of infectious diseases since the pioneer work of Kermack and McKendrick [1], which provides us useful control measures (see e.g. [2,3]). From then on, many famous models of infectious disease population dynamics have been formulated (see e.g. [1,2,4]). One of the famous disease model is SIRS epidemic model (see [2]) which takes the following form

$$\begin{cases} S' = \Lambda - dS - \frac{\beta SI}{N} + \delta R, \\ I' = \frac{\beta SI}{N} - (\gamma + d + \alpha)I, \\ R' = \gamma I - (\delta + d)R, \end{cases} \quad (1.1)$$

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where S is the number of the individuals susceptible to the disease, I denotes the number of the individuals who are infectious and R represents the members who have been removed from the possibility of infection, $N = S + I + R$ denotes the total population individuals, Λ is the influx of individuals into the susceptible; d is the natural death rate of S , I and R compartments; γ denotes the recovery rate of the infective individuals, δ represents the rate at which recovered individuals loss immunity and return to the susceptible class, β denotes the transmission rate and α denotes the death rate due to disease. The parameters involved in system (1.1) are positive constants. In system (1.1), the basic reproduction number is $R_0 = \frac{\beta}{d+\gamma+\alpha}$ which determines the epidemic occurs or not. If $R_0 < 1$, then system (1.1) has only the disease-free equilibrium $E_0 = (S_0, 0, 0) = (\frac{\Lambda}{d}, 0, 0)$ and it is globally asymptotically stable in the invariant set Γ . This shows that the disease will die out and the entire population will be susceptible. If $R_0 > 1$ and $\alpha = 0$, then E_0 is unstable and system (1.1) has a unique positive endemic equilibrium $E^* = (S^*, I^*, R^*)$ which is globally asymptotically stable in the region Γ , where $S^* > 0$, $I^* > 0$, $R^* > 0$ and $\Gamma = \{(S, I, R) : S > 0, I > 0, R > 0, S + I + R \leq \frac{\Lambda}{d}\}$. This means that the disease will prevail and persist in a population.

On the other hand, in the real world, epidemic models are always affected by the environmental noise (see e.g. [5–12]). Stochastic models may be a more appropriate way of modeling epidemics in many circumstances (see e.g. [13–18]). For example, stochastic models are able to take care of randomness of infectious contacts occurring in the latent and infectious periods [19]. It also has been shown that some stochastic epidemic models can provide an additional degree of realism in comparison with their deterministic counterparts (see e.g. [5,20–29]). Especially, Allen et al. [5] revealed that stochastic model should suit the question of disease extinction better. Herwaarden et al. [21] suggested that an endemic equilibrium in a deterministic model can disappear in its corresponding stochastic system due to stochastic fluctuations. And Näsell [22] formulated stochastic models to show that some stochastic models are a better approach to describe epidemics for a large range of realistic parameter values in comparison with their deterministic counterparts.

There are different approaches to introduce random perturbations in the model, both from a mathematical and biological perspective [27,30]. The stochastic SIRS epidemic model introduced in this paper adopts the approach by Mao et al. [9], and assume that the parameters involved in the model always fluctuate around some average value due to continuous fluctuation in the environment. Following this approach, we research a SDE SIRS epidemic model with standard incidence where we assume that the environmental noise is proportional to the variables. Then corresponding to system (1.1), we have the following stochastic model

$$\begin{cases} dS = \left[\Lambda - dS - \frac{\beta SI}{N} + \delta R \right] dt + \sigma_1 S dB_1(t), \\ dI = \left[\frac{\beta SI}{N} - (\gamma + d + \alpha)I \right] dt + \sigma_2 I dB_2(t), \\ dR = [\gamma I - (\delta + d)R] dt + \sigma_3 R dB_3(t), \end{cases} \quad (1.2)$$

where $B_i(t)$ are independent standard Brownian motions with $B_i(0) = 0$ and $\sigma_i^2 > 0$ denote the intensities of the white noise, $i = 1, 2, 3$. Other parameters are the same as in system (1.1).

This paper is organized as follows. In Section 2, we present some preliminaries which will be used in our following analysis. In Section 3, we show that there exists a unique global positive solution of system (1.2). In Section 4, we verify that there is an ergodic stationary distribution of system (1.2). In Section 5, we establish sufficient conditions for extinction of the disease. Finally, we provide a brief discussion and the summary of the main results.

2. Preliminaries

Throughout this paper, let $(\Omega, \mathcal{F}, \mathbb{P})$ be a complete probability space with a filtration $\{\mathcal{F}_t\}_{t \geq 0}$ satisfying the usual conditions (i.e., it is increasing and right continuous while $\mathcal{F}_0$ contains all $\mathbb{P}$ -null sets), $B_i(t)$ ($i = 1, 2, 3$) are defined on this complete probability space, we also let $\mathbb{R}_+^d = \{x \in \mathbb{R}^d : x_i > 0, 1 \leq i \leq d\}$.

In general, consider the d -dimensional stochastic differential equation

$$dX(t) = f(X(t), t)dt + g(X(t), t)dB(t) \quad \text{for } t \geq t_0, \quad (2.1)$$

with initial value $X(0) = X_0 \in \mathbb{R}^d$. $B(t)$ represents an n -dimensional standard Brownian motion defined on the complete probability space $(\Omega, \mathcal{F}, \{\mathcal{F}_t\}_{t \geq 0}, \mathbb{P})$. Denote by $C^{2,1}(\mathbb{R}^d \times [t_0, \infty]; \mathbb{R}_+)$ the family of all nonnegative functions $V(X, t)$ defined on $\mathbb{R}^d \times [t_0, \infty]$ such that they are continuously twice differentiable in X and once in t . The differential operator L of Eq. (2.1) is defined by

$$L = \frac{\partial}{\partial t} + \sum_{i=1}^d f_i(X, t) \frac{\partial}{\partial X_i} + \frac{1}{2} \sum_{i,j=1}^d [g^T(X, t)g(X, t)]_{ij} \frac{\partial^2}{\partial X_i \partial X_j}.$$

If L acts on a function $V \in C^{2,1}(\mathbb{R}^d \times [t_0, \infty]; \mathbb{R}_+)$, then

$$LV(X, t) = V_t(X, t) + V_x(X, t)f(X, t) + \frac{1}{2} \text{trace}[g^T(X, t)V_{xx}(X, t)g(X, t)],$$

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