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Degeneracy estimation in interference models on wireless networks

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HIGHLIGHTS

- A Monte Carlo investigation of interference in real-world wireless networks.
- Degeneracy estimate of interference from spectrum allocation configurations.
- Antiferromagnetic q-state Potts model density of states.
- Wang–Landau algorithm for frustrated spin systems on connected graphs.
- Critical interference levels estimated from density of states CDF.

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ABSTRACT

We present a Monte Carlo study of interference in real-world wireless networks using the Potts model. Our approach maps the Potts energy to discrete interference levels. These levels depend on the configurations of radio frequency allocation in the network. For the first time, we estimate the degeneracy of these interference levels using the Wang–Landau algorithm. The cumulative distribution function of the resulting density of states is found to increase rapidly at a critical interference value. We compare these critical values for several different real-world interference networks and Potts models. Our results show that models with a greater number of available frequency channels and less dense interference networks result in the majority of configurations having lower interference levels. Consequently, their critical interference levels occur at lower values. Furthermore, the area under the density of states increases and shifts to lower interference values. Therefore, the probability of randomly sampling low interference configurations is higher under these conditions. This result can be used to consider dynamic and distributed spectrum allocation in future wireless networks.

1. Introduction

Complex systems are characterised by "large networks of components with no central control and simple rules of operation [that] give rise to complex collective behaviour, sophisticated information processing, and adaptation via learning or evolution" [1]. The treatment of large mobile and wireless communications networks as complex systems provides insights into their collective behaviour and enables the useful application of a wealth of techniques. Modern mobile networks

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are increasingly formed by an ever-growing number of nodes that (due to limits in signalling exchange capacity) must make local decisions in reaction to the surrounding environment while still maintaining global standards of user experience and network performance.

Network science has matured as a field, encompassing both the behaviour of networks themselves and as a substrate for dynamical processes. Increasingly, networks and the dynamics placed on them are being studied by statistical physicists. The critical phenomena of complex networks have been well studied as in Ref. [2]. Here the authors review both structural phase transitions and those of systems where the network and interacting agents influence each other. In Ref. [3], the critical temperature T_C of the antiferromagnetic Ising model is shown to be related to the degree distribution of nodes in scale-free networks. Random network ensembles have also been characterised by the entropy of networks in the ensemble [4].

Recent studies treating mobile networks as complex systems such as in Refs. [5–9] focus on human dynamics and social interaction via mobile phones. Critical phenomena in wired communication networks are studied in Refs. [10–12]. Transmission rate calculation and interference reduction in wireless networks was performed in Refs. [13,14] using the replica method to evaluate an analogue of the free energy of the system. A great deal of attention has especially been given to the Ising and Potts models in wireless networks. Some applications of these spin models have been; calculating the transmission probability and throughput [15,16], recreating network topology based on mutual information shared between nodes [17], distributed configuration management [18] and adaptive scheduling for wireless networks of sensors for energy efficiency [19]. The use of interference graphs to study interference in wireless networks is discussed in Ref. [20]. Alternative methods for statistical interference analysis among the wireless network community also include stochastic geometry of node placement [21] and a novel circular interference model as in Ref. [22].

Our approach examines the physical network infrastructure as a complex system. This work is part of a broad program undertaken by some of the authors to study the information theoretical and dynamical properties of mobile networks as complex systems. The broad goal of this program is to design networks which perform consistently and optimally with a minimal amount of planning, coordination, and human intervention. Previous work in this program introduces a complexity metric that captures the amount of structure present in self-organising networks [23,24]. The authors find that the complex behaviour exhibited by a collection of self-organising wireless networks is robust to changes in the environment.

The current work uses a multicanonical Monte Carlo method, the Wang–Landau algorithm [25]. We investigate degeneracy in a model of interference in cellular networks with realistic deployments and several orthogonal frequency channels. Our model maps the Potts energy [26] embedded on wireless networks to discrete interference levels. Wireless networks are represented as a collection of basestations which interfere only with neighbouring basestations broadcasting on the same frequency channel. The interference levels are calculated from spectrum allocation configurations. Consequently, many spectrum allocation microstates contribute to each interference level macrostate. The degeneracy of these interference levels is numerically estimated to find the density of states (DOS). Using the DOS, we examine the cumulative distribution function (CDF) of interference levels.

This paper is arranged into six sections. Section 2 provides a brief overview of the Potts model used to describe interference levels. Section 3 details the Wang–Landau Monte Carlo algorithm [25] used to estimate the degeneracy of interference levels. The interference graphs are defined in Section 4 by considering the base station deployment and assigning their nearest neighbours. Section 5 presents numerical results using the Wang–Landau algorithm on these interference graphs and Section 6 concludes.

2. Potts model

This section describes the Potts model and maps network interference to the Potts energy. Limits on signalling exchange capacity force modern mobile networks to make local decisions based on the surrounding environment, suggesting that the nearest neighbour interactions in the Potts model provide a simplified but apt description of interference phenomena on these networks.

The Potts model is typically defined on a lattice and specified by the number of possible states at each site (q) as well as the interaction energy (J). We consider the more general situation of the Potts model on networks represented by connected graphs with sets, (V, L), of vertices and edges (also known as links). Each vertex (basestation sector[1]) $i \in V$ possesses a transmitter using a frequency channel (spin) which may take one of the discrete values $\sigma_i \in [1, q]$ representing the division of the spectrum into q orthogonal channels. Each spin interacts with its nearest neighbours only, interfering with all neighbours using the same channel. The local interference per basestation is summed to define the overall interference, which is equivalent to the Potts model energy. Several q values are considered in this paper on three different graphs, derived from a real-world wireless network.

The energy (Hamiltonian) of the Potts models used in this work is given by

$$H(\sigma) = -J \sum_{\langle i,j \rangle} \delta_{\sigma_i \sigma_j}, \tag{1}$$

[1] In cellular networks, a basestation covers a given area which is divided into sectors, each served by a separate transmitter.