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The investigation of the reentrance phenomenon in cellular automaton traffic flow model

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HIGHLIGHTS

- A new injection strategy in the Nagel–Schreckenberg model with open boundaries is proposed.
- A reentrance phenomenon of the low density phase for high injection rates is found.
- The reentrance phenomenon is due to the frustration of the injected vehicle.
- The in-flow and the system capacity explain the observed phase transitions.

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ABSTRACT

We investigate analytically and by numerical simulations, the reentrant low density phase for high injection rate in the Nagel–Schreckenberg model. It is found that the reentrance phenomenon is a direct consequence of our injection strategy. Indeed, by adopting our injection rule, an injection rate exists, above which the in-flow begins to decrease by increasing the injection rate. In addition, we have studied the extraction rate interval at which the reentrance of the low density phase appears. It is found that this interval increases with increasing the maximal velocity. For the non deterministic case, the reentrance interval shifts to higher values of extraction rate.

1. Introduction

Traffic modeling aims at predicting the congestions and to control the circulation efficiently. Indeed, the congestions have disastrous effects on the economy, human health and the environment. In that respect, physicists have proposed various models. Namely, the macroscopic models which consider the traffic as a compressible fluid. The mesoscopic models that consider the traffic as a gas of interacting particles. At last, the microscopic models that track the individual proprieties of vehicles [1].

In the scope of the microscopic models, Nagel and Schreckenberg proposed a cellular automaton model for traffic flow (NaSch model) [2]. It is a simple model that contains the basic traffic rules and exhibits various interesting phenomena. Namely, self organization [3], spontaneous traffic jam and the stop to go waves [4].

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By considering periodic boundaries, more complex phenomena have been considered. In this context, the heterogeneity of traffic was studied [5], the effect of ramps [6], and the effect of defects [7,8]. Periodic boundaries are used to control the vehicles density.

However, the open boundaries are more realistic. Indeed, Popkov et al. [9] have demonstrated empirically how the boundaries induce the phase transitions in traffic flow. It must be noted that those transitions do not exist in the one dimensional equilibrium systems [10]. For an open boundaries traffic flow with a maximal velocity $\text{Vmax} = 1$, the NaSch model becomes identical to the totally asymmetric exclusion process (TASEP). In this context, analytical and simulation studies have been realized [11–14]. The TASEP phase diagram exhibits three phases: the low density phase, the high density phase, and the maximum current phase.

The phase diagram of the NaSch model with $\text{Vmax} > 1$ has also been studied. It is found that the vehicles injection strategy impacts significantly the phase diagram [15–17]. Indeed, some bulk density values are not be achieved by the system in the injection strategy proposed by Cheybani et al. [15]. However, Barlovic et al. [17] established an injection strategy that covers the whole spectrum of density. In the same perspective, Fouladvand et al. [18] proposed an injection strategy that ensures a non accumulation of cars at an intersection with lights.

In this paper, we propose another injection strategy which leads to the reentrance phenomenon. We recall that the reentrance is a noteworthy phenomenon in physical systems. It is exhibited when one phase exists inside some closed temperature range, with a transition to the same phase at low and high temperatures [19]. The reentrance is widely exhibited in magnetic systems [20,21].

In the context of traffic flow, the reentrance phenomenon has not received the attention it deserves. Little works have mentioned it but without giving a clear explanation. In this context, Antal et al. [22] found a reentrance of the high density phase by using attractive interaction in ASEP with next-nearest-neighbor interaction. They explained that as a result of complex interaction between the effective boundaries. By using the Krauss model, Namazi et al. [23] have found a reentrance in their first injection strategy which was explained as a cut off in the maximum flow for an injection rate threshold. Jia et al. [24] have found a reentrance by using the standard injection strategy [15] for high maximal velocities ($\text{Vmax} \geq 5$). However, the physics behind this phenomenon exhibited on such non equilibrium system was not explained.

We believe that a deep investigation of that mechanism is necessary for both theoretical and practical perspectives. For that purpose, we will use the NaSch model, which is a basic model. We will study by analytical methods and by simulations the thresholds of the reentrance appearance. In addition, we will show how the manner of injecting vehicles impacts the reentrance appearance. Those studies will be performed for different maximal velocities for both deterministic and non-deterministic cases.

This paper is organized as follows: first, we will define our injection strategy. After that, we will discuss the results for the deterministic and the non deterministic dynamics. The last section will be devoted to conclusions.

2. Model and injection strategy

In next section, we will briefly describe the cellular automata NaSch model, recall the rule of motion, and explain our open boundaries strategy.

2.1. The NaSch model

The cellular automaton model is an idealization of the physical system. The space, the time and the velocity are discretized variables. The road is a one dimensional chain represented by cells. Each cell can be either empty or occupied by one vehicle.

The NaSch model is characterized by the rule of vehicles movement. At each time step, the system is updated according to the following rules:

Acceleration: $v_n = \min(\text{Vmax}, v_n + 1)$, Vmax is the maximal velocity.
Deceleration: $v_n = \min(v_n, d_n)$, where $d_n = x_{n+1} - x_n - 1$ is the gap between the leading and the preceding vehicle.
Randomization: If $v_n > 0$, $v_n \to \max(v_n - 1, 0)$ with probability P.
Vehicle motion: $x_n \to x_n + v_n$.
x_n and v_n denote respectively, position and velocity of the vehicle n.

2.2. The injection strategy

In the previous investigations of traffic models under open boundaries, the road is modeled by a chain where vehicles enter it from the left end and leave it at the right end (for review see Refs. [15,17]).

At the left end, particles are injected (from an infinite particle reservoir) with a rate α. In this context many strategies of injection were proposed, namely:

- The standard injection strategy, where particles are injected at a cell outside the system with a probability α. In this case, the velocity is based on the distance of the following vehicle on the road. If the velocity is zero, the particle will be deleted [15].