Physica A 469 (2017) 224–231

Contents lists available at ScienceDirect

Physica A

journal homepage: www.elsevier.com/locate/physa

# Effect of periodic inflow on speed-controlled shuttle bus

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## HIGHLIGHTS

- We presented the dynamic model of a speed-controlled bus with the periodic inflow.
- We studied the effect of the periodic inflow on dynamic motion of the speed-controlled bus.
- The bus motion exhibits the periodic, quasi-periodic, and chaotic motions.

## ARTICLE INFO

*Article history:*
Received 18 September 2016
Available online 17 November 2016

*Keywords:*
Shuttle bus
Transportation
Nonlinear map
Quasi-periodic motion
Chaos
Fluctuation
Complex system

## ABSTRACT

We investigate the dynamic behavior of a shuttle bus controlled the speed when passengers come periodically at the origin. We propose the nonlinear-map model for the dynamics of the speed-controlled bus with the periodic inflow. The bus schedule is closely connected to the motion. The motion of the speed-controlled bus is affected by the periodic inflow. The motion of the shuttle bus depends highly on both speed control and periodic inflow. The shuttle bus displays the periodic, quasi-periodic, and chaotic motions by varying both periodic inflow and speed control. We clarify the dependence of the bus motion on both speed control and periodic inflow.



## 1. Introduction

Recently, physicists have investigated transportation problems from a point of view of nonlinear dynamics and statistical mechanics [1–5]. They have applied the concepts and techniques of physics to the transportation systems [6–46]. The dynamic transitions and chaos have been found in the traffic flow, pedestrian flow, and bus transportation [1–3,28,33,40,41]. The dynamic transition and chaos are typical signatures of the complex behavior of transportation system. The dynamic behavior of buses has been studied when the buses shuttle repeatedly between the starting terminal (origin) and the final terminal (destination). It has been found that the shuttle bus displays the complex motion by the interaction between buses and passengers [40,41].

The shuttle bus system exhibits severe congestion problems in the peak traffic. In managing the shuttle bus operation, it is necessary and important to estimate the arrival time of the bus accurately for a bus schedule. Furthermore, it is important to transport passengers from the starting point (a terminal) to his destination (other terminal) within some period of time for the rush hour trips [40,41].

In the public transportation system, it is important and necessary to make the bus schedule. It is well known that passengers using buses are served best when buses arrive at stations on time and there is no congestion. Frequently, buses are delayed or go faster in the bus transport system. It is hard to operate buses on time. The bus schedule is closely related to the dynamic motion of buses. The bus dynamics depends highly on the speed control method. The arrival time of the bus

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http://dx.doi.org/10.1016/j.physa.2016.11.017
0378-4371/

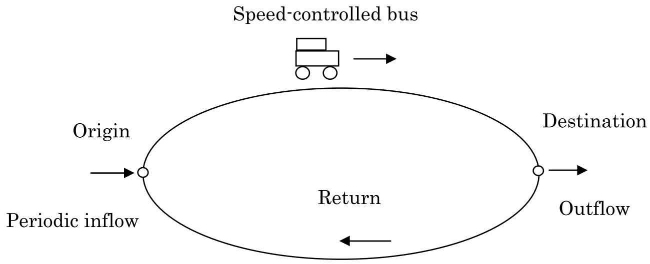


**Fig. 1.** Schematic illustration of the speed-controlled bus shuttled between the origin and the destination. Passengers come into the origin periodically.

is determined by the speed control method of the bus. The dynamics of the speed-controlled bus has been studied [47,48]. It has been shown that the speed-controlled bus displays the complex motion.

In real bus traffic, the inflow rate of passengers into the bus varies greatly with time. The bus systems with a periodic inflow have been studied. It has been shown that the bus displays a complex motion with varying the inflow rate and the period [49]. The dynamic motion of the shuttle bus depends highly on the inflow rate and the period of incoming passengers at the starting terminal. However, it is unknown how does the periodic inflow affect the motion of the speed-controlled bus. It is anticipated that the motion of the bus changes greatly by adding the periodic inflow to the speed-controlled bus. The combined effect of the speed control with the periodic inflow has a important influence on the bus dynamics. The combined effect will induce a complex motion of the bus by interaction between the speed control and the periodic inflow. In the bus transport system, it is necessary and important to make an accurate estimate of the arrival time for a bus schedule.

In this paper, we study the dynamic behavior of the speed-controlled shuttle bus with periodic inflow of passengers. We describe the dynamics of the shuttle bus transport in terms of the nonlinear map. We present an extended version of the circle map for the shuttle bus system. We investigate how the dynamic behavior changes by adding the periodic inflow on the speed-controlled bus. We show that the shuttle bus displays the periodic, quasi-periodic, and chaotic motions in a complex manner by varying both speed control and periodic inflow.

## 2. Nonlinear-map model

We consider the dynamic model of the shuttle bus system which mimics the service of a bus shuttling repeatedly between the origin and the destination (for example, an airport and a railway station). The bus carries the passengers at the origin to the destination. When the tram or airplane arrives at terminals periodically, the passengers come into terminals periodically. Then, passengers board on the bus at terminals with periodic inflows.

We model the public transport system of a shuttle bus as follows. A single bus shuttles repeatedly between the starting point (origin) and the destination. The starting point is the only position to take the bus. The passengers board the bus at the origin and then the bus starts from the origin. The bus moves toward the destination. When the bus arrives at the destination, all riding passengers leave the bus. As soon as all passengers get off the bus, the bus leaves the destination and then returns to the origin. The process is repeated. Fig. 1 shows the schematic illustration of the shuttle bus transport between the origin and the destination.

If the bus is delayed, the bus driver speeds up to retrieve the delay. The driver controls the speed of the bus according to the tour time. The tour time is defined as the time that the bus travels round. The driver speeds up more and more when tour time $\Delta t$ approaches critical value $t_c$ and the bus speed reaches maximum speed $v_{\max}$ after a while. We assume that the speed is controlled by the following function:

$$v(\Delta t) = v_0 + \frac{\tilde{v}}{2}\left\{1.0 + \tanh[a(\Delta t - t_c)]\right\}, \tag{1}$$

where $v_0$ is the cruising speed, $\tilde{v} = v_{\max} - v_0$, and $v_{\max}$ is the maximum speed. Parameter $a$ is the slope of the speed control function at the turning point. It represents the degree of the speed change. If the driver changes the speed abruptly (gradual), parameter $a$ has a high (low) value. When $\Delta t \to \infty$, $v(\Delta t) \to v_{\max}$.

We describe the dynamic model of the bus system in terms of the nonlinear map. We assume that passengers come into the origin periodically with period $t_s$. Define the number of passengers boarding the bus at the origin and trip $n$ by $B(n)$. The parameter $\gamma$ is the time it takes one passenger to board the bus, so $\gamma B(n)$ is the amount of time needed to board all the passengers at the origin. The moving time of the bus between the origin and the destination is $L/v(\Delta t(n))$ where $L$ is the length between the origin and the destination and $v(\Delta t(n))$ is the mean speed of bus at trip $n$. The stopping time at the destination to leave off the passengers is $\beta B(n)$ where parameter $\beta$ is the time it takes one passenger to leave the bus. The tour time equals the sum of these periods. The arrival time $t(n+1)$ of the bus at the origin on trip $n+1$ is given by

$$t(n+1) = t(n) + \gamma B(n) + \beta B(n) + \frac{2L}{v(\Delta t(n))}. \tag{2}$$