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Analysis of social optimum for staggered shifts in a single-entry traffic corridor with no late arrivals

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HIGHLIGHTS

- Effects of staggered shifts on traffic corridor are studied under SO principle.
- Closed-form SO solution is derived and numerical examples are presented.
- Optimum proportion of the numbers of commuters is discussed in detail.
- Cumulative outflow curve under the SO principle is piecewise smooth.

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ABSTRACT

This paper investigates the traffic flow dynamics under the social optimum (SO) principle in a single-entry traffic corridor with staggered shifts from the analytical and numerical perspectives. The LWR (Lighthill–Whitham and Richards) model and the Greenshield's velocity–density function are utilized to describe the dynamic properties of traffic flow. The closed-form SO solution is analytically derived and some numerical examples are used to further testify the analytical solution. The optimum proportion of the numbers of commuters with different desired arrival times is further discussed, where the analytical and numerical results both indicate that the cumulative outflow curve under the SO principle is piecewise smooth.

1. Introduction

In recent years, staggered shifts have attracted researchers to propose some models to study this topic since it has direct effects on commuters' travel behavior (especially their commuting behavior) and partly relieve traffic congestion [1–8]. Roughly speaking, the studies can be divided into two groups, where some use survey data and case study to explore the effects of staggered shifts on commuting behavior [1–3] and others focus on proposing models to explore the effects of staggered shifts on the whole traffic system [4–8]. In addition, most of the above studies are proposed based on the classical bottleneck model [9].

However, the basic bottleneck model [9] treats the traffic congestion as a queue behind a single bottleneck with fixed capacity, so the dynamic properties of traffic stream and traffic jam cannot explicitly be investigated. Hence, the effects of staggered shifts on traffic dynamics cannot explicitly be reproduced in the studies. In fact, the propagation properties of traffic dynamics have prominent influences on commuters' departure time choice, total trip cost, and the time–space

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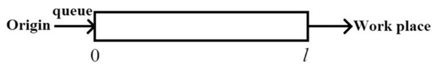

Fig. 1. Schematic diagram of morning commuting in a single-entry traffic corridor.

distribution of traffic flow. This topic has in detail been discussed by Newell [10] (hereinafter referred to as Newell), DePalma and Arnott [11] (hereinafter referred to as DA), Lai et al. [12] and Redhu and Gupta [13,14]. In addition, comparing with the situation that all commuters have the same desired arrival time, staggered shifts may have remarkable effects on traffic dynamic properties since different desired arrival times directly influence commuters' departure time and trip cost.

As for the dynamic properties, researchers proposed many traffic flow models to study the micro driving behaviors [15–19], traffic jam [20–23], and control strategy [24–34]. Nevertheless, these traffic flow models [15–34] cannot directly be used to analyze the effects of staggered shifts on total trip cost. Hence, we in this paper extend the work [10,11] to first investigate the effects of staggered shifts on the flow pattern under the SO principle in a single-entry traffic corridor with no late arrivals, then deduce the optimum proportion of the numbers of commuters in order to minimize the system's total trip cost, finally used numerical tests to further testify the analytical results.

2. Model

In this section, we focus on proposing a SO model for staggered shifts in a single-entry traffic corridor with no late arrivals (see Fig. 1). In this paper, we use the word "departure" to denote a vehicle's departure from origin and consequent entry into the corridor, and the word "arrival" to indicate a vehicle's arrival at destination and consequent exit from the corridor. In addition, we should give the following assumptions:

(1) Vehicles travel along a single-lane road with length l and constant width; the road has only one entry and one exit located at $x = 0$ and $x = l$, respectively.
(2) Point queue may occur at the entry point if the departure rate exceeds the road capacity; the distance between origin and entry point, and the distance between destination and exit point are ignored.
(3) Two desired arrival times are considered, where some commuters (N_1) have the desired arrival time $\bar{t}_1$, other commuters (N_2) have another desired arrival time $\bar{t}_2$, and $\bar{t}_1 < \bar{t}_2$.
(4) Each vehicle's trip cost can here be defined as a linear function of travel time and schedule delay time, i.e.,

$$C(t) = \begin{cases} \alpha\tau(t) + \beta\left(\bar{t}_1 - (t + \tau(t))\right), & t + \tau(t) \le \bar{t}_1 \\ \alpha\tau(t) + \beta\left(\bar{t}_2 - (t + \tau(t))\right), & \bar{t}_1 < t + \tau(t) \le \bar{t}_2, \end{cases} \tag{1}$$

where $\tau(t)$ is the travel time of the vehicle departing at time t (which includes the time spent in a queue if it appears); α is the per unit cost of travel time; β is the per unit cost of early arrival time; $\beta < \alpha$ [35]; $C(t)$ is the trip cost.

Next, we used the LWR (Lighthill–Whitham [36,37] and Richards [38]) model to study the SO solution for staggered shifts in a single-entry traffic corridor with no late arrivals, where the LWR model can be formulated as follows:

$$\frac{\partial k}{\partial t} + \frac{\partial q}{\partial x} = 0, \tag{2}$$

where x denotes location; t denotes time; k, q are respectively the traffic density and flow. For solving the LWR model, an explicit velocity–density function is needed. For simplicity, we use the Greenshield's velocity–density function, i.e.,

$$v = v_0\left(1 - \frac{k}{k_j}\right), \tag{3}$$

where v and k represent velocity and density respectively; v_0 is the free-flow velocity; k_j is the jam density. Substituting $q = v \cdot k$ into Eq. (3) we have:

$$v = \frac{v_0}{2}\left(1 + \sqrt{1 - \frac{q}{q_m}}\right), \tag{4a}$$

$$k = \frac{k_j}{2}\left(1 - \sqrt{1 - \frac{q}{q_m}}\right), \tag{4b}$$

$$w = \frac{v_0}{q'(k)} = \frac{1}{\sqrt{1 - \frac{q}{q_m}}}, \tag{4c}$$

where $q_m = \frac{1}{4}v_0 k_j$ is the capacity flow; w is the reciprocal of the wave velocity $q'(k) = \mathrm{d}x/\mathrm{d}t = v_0\left(1 - \frac{2k}{k_j}\right)$ normalized by free-flow velocity. Fig. 2 shows the velocity–density relation and the flow–density relation, where $k_m = 0.5k_j$. Note: in